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Improving the predictability of real economic activity and asset returns with forward variances inferred from option portfolios[☆]

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ABSTRACT

This paper presents an option positioning that allows us to infer forward variances from option portfolios. The forward variances we construct from equity index options help to predict (i) growth in measures of real economic activity, (ii) Treasury bill returns, (iii) stock market returns, and (iv) changes in variance swap rates. Our yardstick for measuring predictive ability is both individual and joint parameter statistical significance within a market, as well as across a set of markets.

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1. Introduction

Examining the traditional hypotheses about the term structure of default-free interest rates has played a central

part in financial economics (Cox, Ingersoll, and Ross, 1981; Fama and Bliss, 1987; and Campbell and Shiller, 1991). Key to these analyses are forward interest rates and bond returns, which are both based on the discount bond price representation $\mathbb{E}_t^{\mathbb{Q}}(\exp(-\int_t^{t+\tau} r_u du))$, where τ is term to maturity, r_u is spot interest rate, and $\mathbb{E}_t^{\mathbb{Q}}(\cdot)$ is the time- t conditional expectation under the equivalent martingale measure \mathbb{Q} . If claims contingent on the exponential of integrated variance, i.e., on $\exp(-\int_t^{t+\tau} \sigma_u^2 du)$, where σ_u^2 is instantaneous variance of equity index returns, could be valued, then one could exploit the analogy and formulate and empirically investigate a set of hypotheses linked to variance.

In this paper, we contribute by providing a framework for deducing variance-based analogues of forward interest and relating them to variations in economy-wide variables. Specifically, we ask whether forward variances can help forecast real economic activity, Treasury bill and

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bond returns, stock returns, and movements of asset prices that depend on the integrated variance of equity index returns.

We use the work of Carr and Lee (2008) to synthesize the exponential of integrated variance using a strip of European calls and puts, written on the market index. To study the predictability of asset returns and aggregate real economic activity, we employ S&P 500 index options across four maturities to build time series of prices of exponential claims on integrated variance. These prices are expressed in terms of the prices of investable portfolios of options, and hence, variance is tradable.

Crucial to our investigation are forward variances, extracted from the prices of exponential claims of different maturities. Drawing on the analogy with the term structure of interest rates, the forward variances embody expected variances, inferred from particular index option portfolios. We surmise that forward variances are related to future movements in both the financial and the real sector, as perceived in the equity index option market.³

Inference is based on predictive regressions, and the null hypothesis of no predictability is generally rejected, after addressing econometric concerns. Specifically, our conclusions are robust under the Newey and West (1987, 1994) method with optimal lag selection, the Hodrick (1992) approach for computing standard errors after imposing the null of no predictability, and a parametric bootstrap (i.e., Mark, 1995; Cochrane and Piazzesi, 2005; and Amihud, Hurvich, and Wang, 2009). Our yardstick for measuring predictive ability is both individual and joint statistical significance of the slope coefficients within a market, as well as across a set of markets. A simulation exercise indicates that the Hodrick (1992) test offers the correct size in small samples, under empirically relevant assumptions about the data generating process of the predictors, and even when some predictors are allowed to be nearly integrated.

Summarizing the empirical findings, a high level of front-end forward variance is associated with contracting economic activity, as measured by the growth rate of non-farm payroll, industrial production, and other indicators. Our results suggest that forward variances are useful in predicting Treasury bill returns but fail to predict Treasury bond returns. This is the case regardless of whether the Cochrane and Piazzesi (2005) factor is used as an additional predictor.

³ Our study is related to a body of literature that examines predictability in the equity, bond, and currency markets, e.g., Campbell and Shiller (1988), Ferson and Harvey (1991), Cutler, Poterba, and Summers (1991), Bekaert and Hodrick (1992), Fama and French (1993), Mark (1995), Kirby (1997), Cremers (2002), Ferson, Sarkissian, and Simin (2003), Lettau and Ludvigson (2005), Cochrane and Piazzesi (2005), Campbell and Yogo (2006), Ang and Bekaert (2007), Cochrane (2008), Boudoukh, Richardson, and Whitelaw (2008), Goyal and Welch (2008), Bollerslev, Tauchen, and Zhou (2009), Pástor and Stambaugh (2009), Henkel, Martin, and Nardari (2011), and Rapach, Strauss, and Zhou (2010). Contributions to the study of volatility expectations include Day and Lewis (1988), Stein (1989), Harvey and Whaley (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Jorion (1995), Campa and Chang (1995), Andersen, Bollerslev, Diebold, and Labys (2003), Andersen, Bollerslev, and Meddahi (2005), Ait-Sahalia and Mancini (2008), and Mixon (2009).

Furthermore, some of the forward variances help to predict stock market returns in the presence of traditional predictors. The predictability is statistically significant at horizons up to six months. Finally, we find some statistically significant evidence that forward variances predict log changes in variance swap rates.

Going beyond the single-equation testing framework, we examine a system of predictive regressions, which corroborates joint predictability across markets. Overall, forward variances, extracted from index options, appear to be useful predictors of financial and real economic activity variables.

Our empirical findings should be viewed with some caution. The reason is that the construction of monthly forward variances requires index option data with a dense set of strike prices over multiple maturities, which makes us limit our study to the September 1998 to September 2008 sample period.

The paper proceeds as follows. Section 2 outlines the setup underlying the empirical investigation. Section 3 discusses the construction of forward variances. Section 4 presents evidence on the predictability of real economic activity and asset returns. Section 5 concludes the paper.

2. Exponential claims on integrated variance

Here we adopt the modeling setup of Carr and Lee (2008). Let S_t denote the level of the market index at time t and assume that S_t follows a general diffusion process under the equivalent martingale measure.

The instantaneous variance of market index return is given by the adapted process σ_t^2 . In the absence of price jumps, the integrated variance of returns over an interval $[t, t + \tau_n]$ equals the quadratic variation over $[t, t + \tau_n]$, i.e., $\int_t^{t+\tau_n} \sigma_u^2 du = \langle X \rangle_t^{(n)}$. In the empirical section, we use a set of maturities indexed by $n=1,2,3,\dots$, hence the subscript in τ_n . The dynamics of σ_t^2 is left unspecified and the riskfree interest rate r^* is constant.

We employ a special case of the generic exponential claim on integrated variance, considered in Carr and Lee (2008). Based on their Propositions 5.1 and 5.9, we specifically consider the claim with price:

$$H_t^{(t,n)} \equiv e^{-r^* \tau_n} \mathbb{E}^{\mathbb{Q}} \left\{ \exp \left(- \int_t^{t+\tau_n} \sigma_u^2 du \right) \middle| \mathcal{F}_t \right\} \quad (1)$$

$$= e^{-r^* \tau_n} \mathbb{E}^{\mathbb{Q}} \left\{ \sqrt{\frac{8}{7}} \left(\sqrt{\frac{S_{t+\tau_n}}{S_t}} \right) \times \cos \left(\arctan(1/\sqrt{7}) + \frac{\sqrt{7}}{2} \ln \left(\frac{S_{t+\tau_n}}{S_t} \right) \right) \middle| \mathcal{F}_t \right\}, \quad (2)$$

where $\mathbb{E}^{\mathbb{Q}} \{ \cdot | \mathcal{F}_t \}$ denotes \mathcal{F}_t -conditional expectation under the equivalent martingale measure \mathbb{Q} . In the double superscript (t, n) in $H_t^{(t,n)}$, the t reflects the initiation time for the claim, while the n is associated with the time to maturity τ_n . Finally, the subscript t is the time of observation of the price, matching the time index of the information set \mathcal{F}_t . With the assumption that $\tau_0 = 0$, $H_t^{(t,0)} = 1$, by definition.

In terms of the prices of traded call and put options, $H_t^{(t,n)}$ can be written as

$$H_t^{(t,n)} = e^{-r^t \tau_n} + \int_{K > S_t} \omega[K] C_t^{(n)}[K] dK + \int_{K < S_t} \omega[K] P_t^{(n)}[K] dK, \tag{3}$$

where $C_t^{(n)}[K]$ and $P_t^{(n)}[K]$ are the time- t prices of a call and put option written on the market index, respectively, with time to maturity τ_n and strike price K . Moreover, $e^{-r^t \tau_n}$ is the price at time t of a riskfree discount bond with unit face value and maturity τ_n , and

$$\omega[K] = -\frac{8}{\sqrt{14}} \cos\left(\arctan(1/\sqrt{7}) + \frac{\sqrt{7}}{2} \ln\left(\frac{K}{S_t}\right)\right) \frac{1}{\sqrt{S_t} K^{3/2}} \tag{4}$$

is the option positioning.⁴

Consistent with the theoretical arguments in Carr and Lee (2008), the payoff under the expectation in (2) is a function only of the price of the underlying asset, but at the same time replicates the variance-dependent payoff $\exp(-\int_t^{t+\tau_n} \sigma_u^2 du)$. The treatment of Carr and Lee (2008), and its illustration in (3) and (4), reveals that options summarize information about the pricing densities of both the underlying asset and the integrated variance of its return.

Importantly, the expressions (2)–(4) reveal that one can dispense with the need to have a parametric model of stochastic volatility to compute the price $H_t^{(t,n)}$ of an exponential claim on integrated variance. Moreover, the claim contingent on the integrated variance payoff $\exp(-\int_t^{t+\tau_n} \sigma_u^2 du)$ is investable.

We need to stress that the mapping in (2) is predicated on the continuous price process assumption for the market index. Then, the model-free price $H_t^{(t,n)}$ derived in (3) and (4) is based on spanning the equivalent payoff $\sqrt{\frac{8}{7}}(\sqrt{S_{t+\tau_n}/S_t}) \cos(\arctan(1/\sqrt{7}) + \frac{\sqrt{7}}{2} \ln(S_{t+\tau_n}/S_t))$.

Given the often used assumption that price jumps are independent of the underlying diffusion, i.e., as in the jump-diffusion model of Merton (1976), there are reasons to believe, based on the calibrations done, for instance, in Gatheral (2006), Jiang and Tian (2005), and Carr and Wu (2009) for the case of variance swaps, that the impact of jumps on model-free valuation is likely to be small. Nonetheless, as of yet, no model-free formulations are available for $H_t^{(t,n)}$, where the market index level process admits general jump specifications.

Why do we focus attention on the claim with price given in (2), rather than variance swaps? The primary reason is that this special case of exponential claims provides an analogy with the term structure of interest rates. Observe that the price of a riskfree discount bond at time t is given as $B_t^{(n)} \equiv \mathbb{E}_t^{\mathbb{Q}}\{\exp(-\int_t^{t+\tau_n} r_u du)\}$, where τ_n is time to maturity, and r_u is the stochastically varying spot interest rate. Based on $B_t^{(n)}$, fundamental objects are the forward interest rates, as in, among others, Fama and Bliss (1987) and Campbell and Shiller (1991). In the spirit of this literature, we use the prices $H_t^{(t,n)} = e^{-r^t \tau_n} \mathbb{E}_t^{\mathbb{Q}}\{\exp(-\int_t^{t+\tau_n} \sigma_u^2 du) | \mathcal{F}_t\}$ to define the

forward variance at time t between $t + \tau_{n-1}$ and $t + \tau_n$, which we write as

$$f_t^{(n)} \equiv \ln H_t^{(t,n-1)} - \ln H_t^{(t,n)}, \quad n = 1, 2, 3, \dots, \tag{5}$$

thus formalizing forward variances, based on simultaneously observed at time- t prices of exponential claims on integrated variance of differing maturities.

While we only consider the price $H_t^{(t,n)}$ of the claim at the initiation time t , the price $H_{t+\nu}^{(t,n)}$ of the same claim at any time $t + \nu$ for $\nu \in [0, \tau_n]$ is also available from Carr and Lee (2008). For example, when $\nu = \tau_n - \tau_{n-1}$, we obtain

$$H_{t+\nu}^{(t,n)} = (e^{-\int_t^{t+\nu} \sigma_u^2 du}) e^{-r^t(\tau_n - \nu)} \mathbb{E}_t^{\mathbb{Q}}\{e^{-\int_{t+\nu}^{t+\tau_n} \sigma_u^2 du} | \mathcal{F}_{t+\nu}\} \\ = (e^{-\int_t^{t+\nu} \sigma_u^2 du}) H_{t+\nu}^{(t+\nu, n-1)}. \tag{6}$$

The first expression in (6) contains two valuation components: (i) the exponent $e^{-\int_t^{t+\nu} \sigma_u^2 du}$, reflecting the portion of integrated variance which has been revealed between times t and $t + \nu$, and (ii) the discounted expectation of the remaining uncertainty, which is equal to the price of a new exponential claim on integrated variance, initiated at time $t + \nu$. Therefore, if the security $H_t^{(t,n)}$ is liquidated at time $t + \nu$, it entitles the holder to the value of the claim due to the revealed portion of integrated variance between times t and $t + \nu$, plus the value of a newly initiated at time $t + \nu$ exponential claim on integrated variance with the same maturity date (i.e., with remaining time to maturity $\tau_n - \nu = \tau_{n-1}$).

Eq. (6) is essential for constructing the return of exponential claims on integrated variance with original time to maturity τ_n , given by $1 + h_{t+\nu}^{(n)} \equiv H_{t+\nu}^{(t,n)} / H_t^{(t,n)} = \left(e^{-\int_t^{t+\nu} \sigma_u^2 du}\right)$

$H_{t+\nu}^{(t+\nu, n-1)} / H_t^{(t,n)}$. The term $\int_t^{t+\nu} \sigma_u^2 du$ reflects the revealed portion of integrated variance between $[t, t + \nu]$, and has no counterpart in the expression for the return of a riskfree discount bond. Thus, we recognize that the nature of the exponential claim on integrated variance differentiates it from a discount bond, and brings deviation from a complete analogy between the expressions for their respective returns.

The forward variances, as defined in (5), can be used to formulate and test empirical hypotheses about the predictability of real economic activity and financial asset returns. One advantage of forward variances in this context is that they are derived from the prices $H_t^{(t,n)}$ of investable claims.

In particular, $H_t^{(t,n)}$ are the prices of portfolios of traded call and put options, as given in (3) and (4). Our empirical approach thus exploits the investability of the assets involved, much in the same way as the empirical specifications of Fama and Bliss (1987), Campbell and Shiller (1991), Bekaert, Hodrick, and Marshall (1997), Backus, Foresi, Mozumdar, and Wu (2001), Dai and Singleton (2002), and Cochrane and Piazzesi (2005) are based on traded bond prices.

3. Forward variances from option portfolios

What is special about the investable option portfolios that underlie the exponential claims on integrated variance is that they incorporate a pure volatility risk component and are

⁴ The SSRN version of our paper contains the steps of the proof of Eq. (4) (Bakshi, Panayotov, and Skoulakis, 2010).

devoid of the market return risk component. Such a feature makes our empirical analysis distinct from the extant literature on option returns. Furthermore, when variances are tradable, it becomes difficult to defend the use of model-based implied volatilities in empirical work.⁵ Exponential claims on integrated variance also stand in contrast to variance swaps in an important way. While the industry practice is similar in inferring variance swap rates from option portfolios, the swaps satisfy zero cost at entry.

In our empirical investigation, we employ S&P 500 index options to construct the prices $H_t^{(t,n)}$. The S&P 500 index option data, obtained from OptionMetrics, are sampled at the end of the month, from September 1998 to September 2008, for a total of 121 months. Given the expiration calendar at the Chicago Board of Options Exchange (CBOE), options with maturity of about 19, 49, and 79 days are available at the end of each month, and prices of options with maturity of about 109 days are constructed, using interpolation when necessary. There are more puts than calls, and relatively more short-maturity options, with a total of 11,490 index options. The data are described in Appendix A.

We use out-of-the-money index calls and puts to calculate the prices $H_t^{(t,n)}$ via (3) and (4) at the end of each month. The four maturities employed correspond roughly to 19, 49, 79, and 109 days, and are indexed by $n=1,2,3,4$ and denoted by τ_n . Thus, we obtain time series of $H_t^{(t,n)}$, one for each maturity,

$$H_t^{(t,n)} \quad \text{for } t = 1, \dots, T = 121, \text{ and } n = 1, 2, 3, 4. \quad (7)$$

The claims with these prices (i) are investable, and (ii) can be represented by traded portfolios of calls and puts. Spanning payoffs with European options, as used in the calculation of $H_t^{(t,n)}$, is reasonably accurate and yields small approximation errors, a point also made by Dennis and Mayhew (2002), Jiang and Tian (2005), and Bakshi and Madan (2006).

The prices $H_t^{(t,n)}$ are the basis for building the time series of forward variances. Specifically, let

$$\mathbf{f}_t \equiv [y_t^{(1)} \quad f_t^{(2)} \quad f_t^{(3)} \quad f_t^{(4)}], \quad (8)$$

where $y_t^{(1)} \equiv -\ln H_t^{(t,1)}$, $f_t^{(2)} \equiv \ln H_t^{(t,1)} - \ln H_t^{(t,2)}$, $f_t^{(3)} \equiv \ln H_t^{(t,2)} - \ln H_t^{(t,3)}$, and $f_t^{(4)} \equiv \ln H_t^{(t,3)} - \ln H_t^{(t,4)}$. Observe that, as in the literature on the term structure of interest rates, $y_t^{(1)}$ can be considered as a forward variance over $[0, \tau_1]$, since $H_t^{(t,0)} = 1$ by definition, and so one could write $y_t^{(1)} = \ln H_t^{(t,0)} - \ln H_t^{(t,1)}$. However, the analogy has its limitation, since, in the term structure context, $y_t^{(1)}$ is the yield at time t of a discount bond with maturity τ_1 . In contrast, the payoff of the security with price $H_t^{(t,1)}$ is random and contingent upon the realized integrated variance until maturity, hence, no yield counterpart can be meaningfully determined at time t . The integrity of the forward variances is evaluated in the data by verifying that

$\mathbb{E}^{\mathbb{Q}} \left\{ - \int_t^{t+\tau_n} \sigma_u^2 du \mid \mathcal{F}_t \right\} \leq \ln \mathbb{E}^{\mathbb{Q}} \left\{ \exp \left(- \int_t^{t+\tau_n} \sigma_u^2 du \right) \mid \mathcal{F}_t \right\}$, which is a consequence of Jensen's inequality.

4. Improving predictability using forward variances

The purpose of this section is to relate proxies for forward variance, as inferred from the option market, to subsequent returns of financial assets and growth in real economic activity. By investigating these relations, we address the pertinent question whether a set of forward variances have the ability to forecast variables that have received attention in prior research, as in, for example, Fama (1984, 1990) and Fama and French (1989, 1993). More importantly, such an investigation allows us to evaluate the information content of forward variances across markets, while accounting for the effect of traditional predictors. In particular, we show the predictive power of the option-based forward variances, and examine whether they improve forecasts of asset returns and real economic activity growth, relative to the earnings yield and term structure of interest rate variables.

4.1. Empirical setup and associated inference concerns

At the heart of our empirical investigation is the following predictive regression for a generic variable Y_{t+1} :

$$Y_{t+1} = \alpha + \beta' \mathbf{f}_t + \theta' \mathbf{Z}_t + \varepsilon_{t+1}, \quad (9)$$

where \mathbf{f}_t is a vector of forward variances defined in (8), and \mathbf{Z}_t is a vector of additional predictors. Following Hodrick (1992), Cochrane and Piazzesi (2005), and Ang and Bekaert (2007), our analysis is focused on the test of joint significance of the slope coefficients. Germane to our goals, we also test the hypothesis that individual forward variances have marginal predictive content for Y_{t+1} , whereby we are careful to address potential concerns about inference.

Given that theory offers little guidance on the directional impact of forward variances on Y_{t+1} , we rely on *two-sided* p -values for the slope coefficients throughout. In the ensuing empirical work, the dependent variable Y_{t+1} in (9) is monthly growth rate or excess return, and, with this feature in mind, all predictors are scaled accordingly. When we expand the scope of the study to consider joint predictability across markets, Y_{t+1} constitutes a low-dimensional vector of growth rates or excess returns and, hence, estimation is based on a system of equations.

When examining inference in the context of predictive specifications of the type in (9), the existing literature has been concerned with two key statistical issues. First, are the predictors stationary? Second, if some predictors display near-unit-root behavior (Cavanagh, Elliott, and Stock, 1995; Valkanov, 2003; Lewellen, 2004; Campbell and Yogo, 2006), then how can one remedy the impact on inference, particularly in small samples?

Small samples can be cause for concern, as our time series have a total of 121 monthly sampled observations. The statistical properties of the forward variances are highlighted in Panels A to E of Table 1, and discussed in some detail in Appendix B.

Whether forward variances exhibit stationarity bears fundamentally on the validity of the predictive regression

⁵ Specifically, it is known that Dupire (2004)'s theory of volatility may be hard to implement for empirical purposes, as local volatility is not tradable. In the same way, results based on stochastic implied volatility may be less than desirable, due to potential model misspecification and the fact that implied volatility is not tradable.

Table 1

Statistical features of forward variances.

Panel A displays cross-correlations between the forward variances, while Panel B presents the autocorrelations up to lag 6. Panel C shows (i) the test statistic for the unit root test of Phillips and Perron (1988), and (ii) the p -values from the unit root test of Parker, Paparoditis, and Politis (2006), based on the stationary bootstrap. For the Phillips-Perron test, the critical values corresponding to p -values of 0.01, 0.05, and 0.10 are -3.46 , -2.88 , and -2.57 , respectively. Panel D reports parameter estimates of the ARMA(1,1)-GARCH(1,1) model using maximum likelihood. This model is selected among ARMA(p, q) and ARMA(p, q)-GARCH(1,1) models, for low orders of p and q , based on the BIC criterion. Finally, Panel E presents the asymptotic p -values for the null hypothesis that the AR coefficient is equal to a value between 0.96 and 0.99, against the one-sided alternative. The sample period is 09/1998 to 09/2008 (121 observations).

	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$
<i>Panel A: Cross-correlations</i>				
$y_t^{(1)}$	1.00			
$f_t^{(2)}$	0.81	1.00		
$f_t^{(3)}$	0.65	0.66	1.00	
$f_t^{(4)}$	0.59	0.63	0.32	1.00
<i>Panel B: Autocorrelations</i>				
ACF(1)	0.70	0.72	0.37	0.61
ACF(2)	0.54	0.67	0.37	0.50
ACF(3)	0.54	0.72	0.61	0.66
ACF(4)	0.43	0.59	0.24	0.56
ACF(5)	0.43	0.58	0.32	0.54
ACF(6)	0.45	0.59	0.49	0.65
<i>Panel C: Unit root tests</i>				
Phillips-Perron: null is $I(1)$	-10.12	-4.25	-8.44	-5.49
Parker-Paparoditis-Polititis: null is $I(1)$, [p-val]	[0.001]	[0.000]	[0.000]	[0.000]
<i>Panel D: ARMA(1,1)-GARCH(1,1) model parameter estimates</i>				
ARMA(1,1)				
Const.	0.000	0.000	0.000	0.000
AR	0.890	0.927	0.874	0.908
MA	-0.220	-0.527	-0.609	-0.667
GARCH(1,1)				
Const.	0.000	0.000	0.000	0.000
ARCH	0.238	0.108	0.340	0.209
GARCH	0.830	0.790	0.679	0.774
<i>Panel E: Near-unit-root behavior under ARMA(1,1)-GARCH(1,1)</i>				
p -val, H_0 : AR coef.=0.96 vs H_a : AR coef. < 0.96	0.091	0.186	0.075	0.146
p -val, H_0 : AR coef.=0.97 vs H_a : AR coef. < 0.97	0.064	0.122	0.054	0.104
p -val, H_0 : AR coef.=0.98 vs H_a : AR coef. < 0.98	0.043	0.075	0.038	0.072
p -val, H_0 : AR coef.=0.99 vs H_a : AR coef. < 0.99	0.028	0.044	0.026	0.048

framework, and we first examine this issue by plotting the forward variances in Fig. 1. We can see that forward variances were about the same in November 1998 as they were in February 2008, a feature unlikely to be associated with a random walk series (Cochrane, 1991a). The stationarity of forward variances also conforms with economic reasoning and modeling practice: if the instantaneous variance is postulated to be a mean-reverting process (e.g., Andersen, Bollerslev, Diebold, and Labys, 2003; and Carr and Lee, 2008), and if exponential claims on integrated variances are valued in that setting, then forward variances inherit stationarity (see a corresponding point in Amihud, Hurvich, and Wang, 2009; and Santos and Veronesi, 2006). In addition, Panel C of Table 1 shows that tests reject the null of unit root for each forward variance, as discussed in Appendix B.

Going further, we observe that the decay of the autocorrelations in Panel B of Table 1 is slower than what can be explained through a purely autoregressive model. Instead, this pattern could be consistent with an

ARMA(1,1) type underlying process, with a sizable negative moving average (MA) component, distinguishing our forward variances from the autoregressive predictors considered in the existing literature. Indeed, information criteria, such as the Bayesian Information Criterion (BIC), suggest that an ARMA(1,1) model with GARCH(1,1) disturbances provides a reasonable parametric model for each of the forward variance series (see also Meddahi, 2003; Ait-Sahalia and Mancini, 2008). Fitting such a model, we obtain AR coefficients in the range 0.874 and 0.927, and MA coefficients in the range -0.220 to -0.667 (see Panel D of Table 1). Whereas near-unit-root behavior by some of the predictors could be an issue, given the estimated coefficients, Panel E of Table 1 provides evidence based on the null hypothesis that the AR coefficient is equal to 0.96, 0.97, 0.98, and 0.99 versus the one-sided alternative, which helps alleviate this concern (see the details in Appendix B).

While univariate regressions featuring a single autoregressive predictor with near-unit-root have been widely

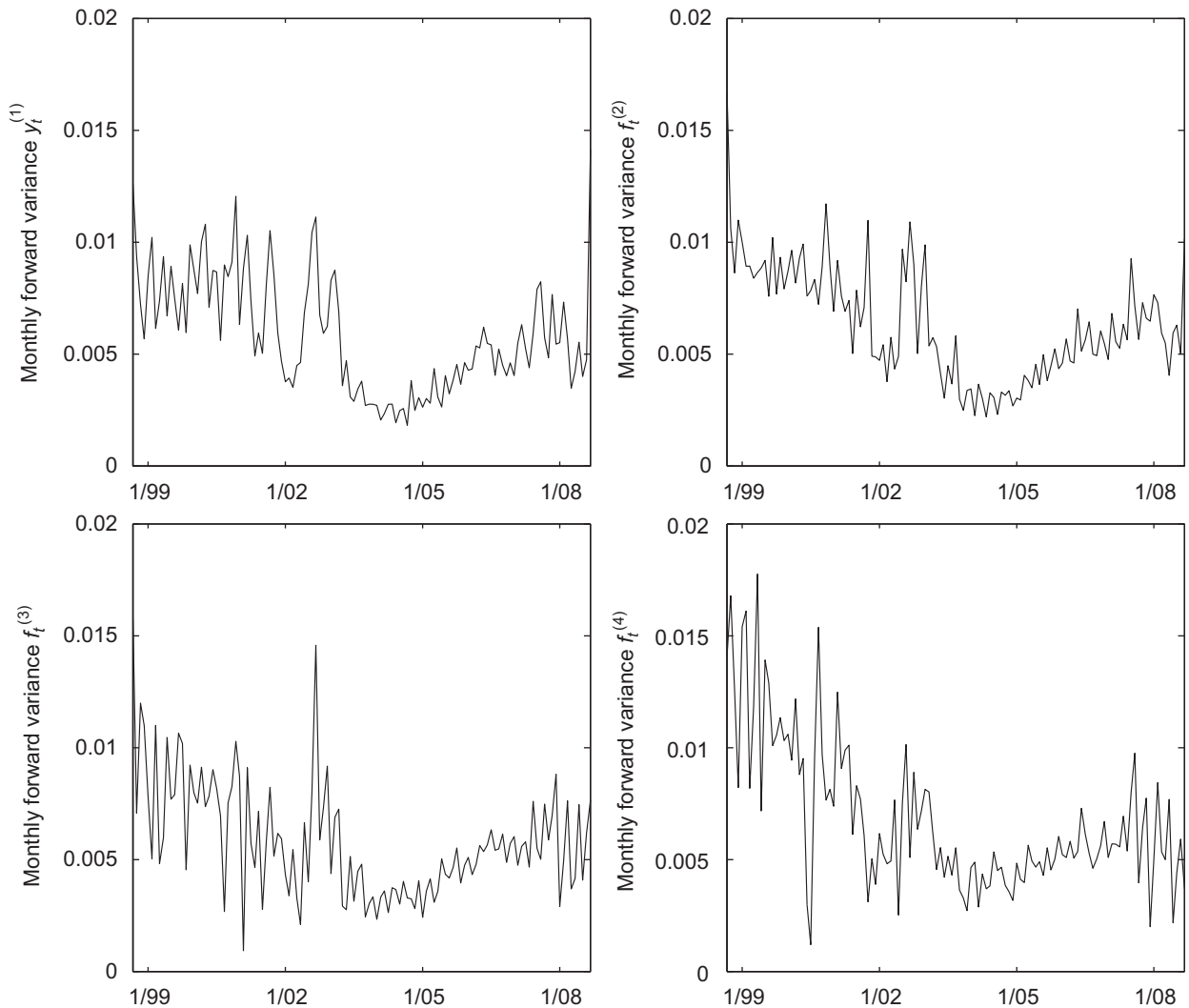


Fig. 1. Time-series behavior of forward variances. At the end of each month over the sample period 09/1998 to 09/2008 (121 observations), we use S&P 500 index options to construct prices of exponential claims on integrated variance, $H_t^{(t,n)}$, where $n=1,2,3,4$ correspond to maturities τ_n of approximately 19, 49, 79, and 109 days, respectively. As in Eqs. (3) and (4), $H_t^{(t,n)} = e^{-r\tau_n} + \int_{K>S_t} \omega[K]C_t^{(n)}[K] dK + \int_{K<S_t} \omega[K]P_t^{(n)}[K] dK$, $\omega[K] = -\frac{8}{\sqrt{14}} \cos(\arctan(1/\sqrt{7}) + \frac{\sqrt{2}}{2} \ln(K/S_t)) / \sqrt{S_t} K^{3/2}$, where $e^{-r\tau_n}$ is the price at time t of a riskfree discount bond with unit face and maturity τ_n , and $C_t^{(n)}[K]$ and $P_t^{(n)}[K]$ are the prices of a call and put with strike price K and maturity τ_n . The forward variances are defined as: $y_t^{(1)} \equiv -\ln H_t^{(t,1)}$, $f_t^{(2)} \equiv \ln H_t^{(t,1)} - \ln H_t^{(t,2)}$, $f_t^{(3)} \equiv \ln H_t^{(t,2)} - \ln H_t^{(t,3)}$, $f_t^{(4)} \equiv \ln H_t^{(t,3)} - \ln H_t^{(t,4)}$. For consistency with the 30-day forward variances $f_t^{(2)}$, $f_t^{(3)}$, and $f_t^{(4)}$, and with the predicted asset returns and real activity variables, which are measured over a monthly interval, the $y_t^{(1)}$ variance is converted to a 30-day rate.

studied,⁶ our focus is on multiple predictors, possibly with a negative MA component. To address the specifics of our problem, we draw on recent treatments which have shown that the Hodrick (1992) 1B covariance matrix estimator can be reliable when the predictors are highly persistent, even in small samples. We mention, among others, Ang and Bekaert (2007, Section 5.2 and footnote 3)

⁶ See, among others, Stambaugh (1999), Valkanov (2003), Torous, Valkanov, and Yan (2004), Lewellen (2004), Campbell and Yogo (2006), and Boudoukh, Richardson, and Whitelaw (2008). Empirical approaches in a multiple predictor setting have been developed in Polk, Thompson, and Vuolteenaho (2006) and Amihud, Hurvich, and Wang (2009), who employ vector autoregressive modeling.

and Wei and Wright (2009). Motivated by these studies, we adopt the Hodrick (1992) estimator as an integral component of inference. In Section 4.6, we provide simulation evidence indicating that the Hodrick (1992) test for joint parameter insignificance offers the correct size, in our setting with ARMA(1,1)-GARCH(1,1) underlying predictors, even when they display near-unit-root behavior.

To further guard against both small sample and possible disturbance distribution misspecification issues, we implement a bootstrap estimation in the spirit of Amihud, Hurvich, and Wang (2009), Cochrane and Piazzesi (2005), and Mark (1995), the details of which are provided in Appendix C, and the results are discussed in Section 4.7. The parametric bootstrap is designed to accommodate the

features of our data, such as a sizeable negative MA component and GARCH effects.

In order to gain broader understanding of the economic relevance of forward variances, we consider several related questions. In particular, do forward variances contain information that helps predict (i) real economic activity, (ii) returns of Treasury bills and bonds, (iii) stock market index returns, and (iv) movements in variance swap rates? The link to a range of financial assets and real economic activity serves to assess whether forward variances reflect developments throughout the economy and beyond the index option market. Such a focus also allows us to gauge whether forward variances capture the dynamics of business conditions over and above traditional predictors, such as earnings yield, slope of the Treasury yield curve, credit spread, and the [Cochrane and Piazzesi \(2005\)](#) factor. Hence, the findings from this part of the investigation can be viewed as complementary to those from, among others, [Fama \(1981, 1990\)](#), [Fama and French \(1989, 1993\)](#), [Cochrane \(1991b\)](#), and [Ilmanen \(1995\)](#). Contributing to the literature, our study accentuates the linkages between financial markets and real economic activity by examining joint predictability through forward variances.

4.2. Forward variances improve predictability of real economic activity

To address the core issues regarding real economic activity (as in, for example, [Lucas, 1977](#); [Stock and Watson, 1989, 2003](#); [Cochrane, 1991b](#); [Schwert, 1990](#)), we specialize (9) to the following:

$$\begin{aligned} g_{t+1}^{\text{payroll}} &= \alpha + \beta' \mathbf{f}_t + \theta_1 \text{yslope}_t + \varepsilon_{t+1}, \\ g_{t+1}^{\text{indus prod}} &= \alpha + \beta' \mathbf{f}_t + \theta_1 \text{yslope}_t + \varepsilon_{t+1}, \end{aligned} \quad (10)$$

where proxies for real activity are initially taken to be monthly growth rates of non-farm payroll and industrial production. The predicted variables are respectively denoted as g_{t+1}^{payroll} and $g_{t+1}^{\text{indus prod}}$ (data source: Bureau of Labor statistical releases and Federal Reserve Board). Estimation results are displayed in [Table 2](#).

Our motivation for considering non-farm payroll and industrial production is that they are summary measures of employment and real output, respectively, and often an integral component of many coincident economic indexes (see, for instance, [Stock and Watson, 2003](#); [Aruoba, Diebold, and Scotti, 2009](#)). Still, we complement our analysis using other measures of real economic activity, including a coincident economic index, and these estimation results are displayed in [Table 3](#).

Besides the forward variances, another predictor employed in Eq. (10) is the slope of the Treasury yield curve, denoted by yslope_t and measured by the difference between the ten-year and the three-month Treasury yields ([Estrella and Hardouvelis, 1991](#); [Ilmanen, 1995](#)). Results with an alternative set of predictors are reported in [Table A1](#), to ascertain the robustness of our conclusions.

Taking into consideration potential econometric concerns associated with the time-series properties of the predictors and small samples, we report the p -values for the regression

coefficients in three ways. First, we compute two-sided p -values based on the Newey and West heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator (denoted as NW-lag, p -val). Our procedure relies on the Bartlett kernel and no prewhitening, and the lag is selected automatically, as in [Newey and West \(1994\)](#). Second, following [Ang and Bekaert \(2007\)](#), [Bollerslev, Tauchen, and Zhou \(2009\)](#), [Chen and Zhang \(2011\)](#), and [Wei and Wright \(2009\)](#), we report two-sided p -values based on the [Hodrick \(1992\)](#) 1B standard errors, under the null of no predictability (denoted as H, p -val). Last, we provide two-sided p -values obtained via a parametric bootstrap (denoted as PB, p -val) using 50,000 bootstrap draws. This bootstrap serves as an additional safeguard in assessing the significance of forward variances, when the sample size is small.

Estimation results presented in Panels A and B of [Table 2](#) establish several aspects of our data. First, a higher value of $y_t^{(1)}$ is associated with a subsequent decline in real economic activity. Based on the estimated p -values (irrespective of the method), the effect is statistically significant at conventional levels for *both* indicators of real activity. Second, higher values of $f_t^{(3)}$ and $f_t^{(4)}$ are associated with improving future real activity, whereby the effect is statistically significant for non-farm payroll with $f_t^{(3)}$, and for both non-farm payroll and industrial production with $f_t^{(4)}$. Third, tabulated results show that forward variances continue to predict real economic activity in the presence of yslope_t . Gauging economic sensitivity, a change of one standard deviation in $y_t^{(1)}$, given the estimated coefficient of -1.080 , translates into a $(-1.080)(0.0026) = -0.281\%$ decline in monthly industrial production. Fourth, $y_t^{(1)}$ and $f_t^{(4)}$ are uniformly significant in each of the predictive regressions.

Moving to the χ^2 test of joint parameter insignificance, the last column of [Table 2](#) shows that three out of four [Hodrick](#) p -values are less than 0.05, implying that the null hypothesis of no predictability is rejected for our measures of real economic activity. This attribute is also detected by the parametric bootstrap. The Newey and West joint p -values are generally lower than their [Hodrick](#) counterparts.

The statistical significance of $y_t^{(1)}$ survives, in our sample, the addition of other yield-based term structure predictors ([Campbell and Ammer, 1993](#); [Ilmanen, 1995](#)) and return-based term structure predictors (see, among others, [Fama and French, 1993](#); [Fung, Hsieh, Naik, and Ramadorai, 2008](#)). For instance, [Table A1](#) illustrates that the importance of $y_t^{(1)}$ is preserved when the yield curve slope is replaced by the credit spread (denoted credit_t), taken to be the spread between Moody's Baa yield and the ten-year Treasury yield.

Collectively, the results in [Tables 2](#) and [A1](#) reveal that credit spread and slope of the yield curve help to predict non-farm payroll, while the slope of the yield curve is insignificant with industrial production. Observe also in [Table 2](#) that the Durbin-Watson (DW) statistics are below 1.39, suggesting autocorrelation in the residuals. Inspection of [Tables 2](#) and [A1](#) furthermore shows that the predictive regressions yield an adjusted R^2 (denoted as \bar{R}^2) between 23% and 44% for non-farm payroll, and between 2% and 7% for industrial production, implying that our

Table 2

Predicting real economic activity as measured by non-farm payroll and industrial production.

This table shows the results from regressions of growth in measures of real economic activity on prior month forward variances and other predictors. The real activity measures are non-farm payroll and industrial production (data source: Bureau of Labor statistical releases and Federal Reserve Board). Reported are coefficient estimates and two-sided p -values for the following regressions over the sample period 09/1998 to 09/2008 (121 observations):

$$g_{t+1}^{\text{payroll}} = \alpha + \beta' \mathbf{f}_t + \theta_1 \text{yslope}_t + \varepsilon_{t+1}, \quad g_{t+1}^{\text{indus prod}} = \alpha + \beta' \mathbf{f}_t + \theta_1 \text{yslope}_t + \varepsilon_{t+1},$$

where \mathbf{f}_t is a vector of forward variances, and g_{t+1} is growth in the real activity variable to be predicted. yslope_t denotes the slope of the Treasury yield curve, defined as the ten-year Treasury yield minus the three-month Treasury yield (data source: Federal Reserve Board). To correct for autocorrelation and heteroskedasticity, we use the Newey-West estimator with automatically selected lag, as in Newey and West (1994), and the reported two-sided p -values are denoted as NW-lag, p -val. Shown also are Hodrick (1992) two-sided p -values (denoted as H, p -val), and two-sided p -values from the parametric bootstrap (denoted as PB, p -val). Adjusted R^2 is reported as \bar{R}^2 , and the Durbin-Watson statistic is shown as $|DW|$. The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. Robustness of the results to alternative choices of the predictor, additional to forward variances, is established in Table A1. All variables are measured in monthly units.

Dependent variable		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	yslope _{<i>t</i>}	\bar{R}^2 DW	Joint p -val
<i>Panel A: Non-farm payroll</i>									
g_{t+1}^{payroll}	Coef.	0.000	-0.409	0.018	0.224	0.218		23.09%	
	NW-8, p -val	0.816	0.000	0.759	0.000	0.000		[0.87]	0.000
	H, p -val	0.723	0.000	0.877	0.004	0.001			0.002
	PB, p -val	0.714	0.000	0.864	0.000	0.000			0.000
g_{t+1}^{payroll}	Coef.	0.001	-0.417	-0.020	0.199	0.198	-0.412	31.64%	
	NW-8, p -val	0.017	0.000	0.742	0.000	0.000	0.001	[0.99]	0.000
	H, p -val	0.004	0.000	0.863	0.011	0.002	0.000		0.000
	PB, p -val	0.007	0.000	0.851	0.005	0.000	0.000		0.000
<i>Panel B: Industrial production</i>									
$g_{t+1}^{\text{indus prod}}$	Coef.	0.000	-1.081	0.473	0.178	0.367		2.60%	
	NW-6, p -val	0.775	0.001	0.128	0.449	0.001		[1.38]	0.000
	H, p -val	0.763	0.006	0.216	0.529	0.032			0.048
	PB, p -val	0.818	0.022	0.184	0.515	0.016			0.063
$g_{t+1}^{\text{indus prod}}$	Coef.	0.000	-1.080	0.480	0.182	0.371	0.070	1.76%	
	NW-6, p -val	0.850	0.001	0.136	0.443	0.002	0.885	[1.38]	0.000
	H, p -val	0.880	0.006	0.206	0.531	0.035	0.888		0.087
	PB, p -val	0.874	0.022	0.187	0.528	0.021	0.899		0.077

predictors do anticipate non-farm payroll data.⁷ Overall, the predictability of real economic activity by some of the forward variances appears to be a robust finding in our sample.

In closing, we ask whether forecastability by forward variances extends to other measures of real economic activity. We consider the Institute of Supply Management's manufacturing index (NAPM), capacity utilization, and civilian unemployment rate, together with retail sales and housing starts (e.g., Beber and Brandt, 2006). Robustness is also assessed using the Conference Board coincident economic index (from Datastream). One result from Table 3 is that retail sales and housing starts are associated with high Hodrick joint p -values, even if $f_t^{(3)}$ has marginal predictive content for retail sales. Another result is that the coefficient on $y_t^{(1)}$ is negative, like before, except for civilian unemployment, which is countercyclical. The forward variance $y_t^{(1)}$ is

generally significant, which corresponds with Table 2 and confirms that forward variances facilitate prediction of a range of real economic activity variables.

4.3. Forward variances track Treasury bill returns, but not Treasury bond returns

Building on Fama and Bliss (1987), Campbell and Shiller (1991), Campbell and Ammer (1993), Ilmanen (1995), and Fama and French (1993), we ask next whether excess returns of Treasury bills and bonds are predictable through forward variances. In analogy to the specification in Eq. (10), we initially consider the predictive regression

$$\ln(1 + r_{t+1}^{\text{treas market}}) - \ln(1 + r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 \text{credit}_t + \theta_2 \text{yslope}_t + \varepsilon_{t+1}, \quad (11)$$

where r_t^* is the one-month riskfree rate, and $r_{t+1}^{\text{treas market}}$ represents r_{t+1}^{bills} and r_{t+1}^{bonds} , which are the total monthly returns of Treasuries with maturity less than one year, and greater than ten years, respectively. The data source is Morningstar (previously Ibbotson Associates). Later, we also examine the role of the Cochrane and Piazzesi (2005) factor.

The specification in (11) could be extended to include additional predictors, and we explain shortly that similar results can be garnered when using alternative term

⁷ We recognize that the R^2 statistics can be biased in small samples. With this in mind, we have examined the empirical distribution of adjusted R^2 in a bootstrap simulation, and we find that (i) the distribution is right-skewed, and (ii) the median is not too different from the reported adjusted R^2 . To save on space, this part of our analysis is omitted. Our objective is to evaluate predictive ability via the χ^2 statistics, and not by the magnitude of the goodness-of-fit statistics across various specifications.

Table 3

Predicting a set of additional real economic activity indicators.

This table shows the results from regressions of real economic activity indicators on prior month forward variances, where the dependent variable is the logarithmic growth rate. We consider (i) Conference Board coincident economic index, (ii) NAPM, (iii) capacity utilization, (iv) civilian unemployment, (v) retail sales, and (vi) housing starts. The source of the Conference Board coincident economic index is Datastream, while other macroeconomic series are taken from the Federal Reserve Bank of St. Louis. To correct for autocorrelation and heteroskedasticity, we use the Newey-West estimator with automatically selected lag, as in Newey and West (1994), and the reported two-sided p -values are denoted as NW-lag, p -val. Shown also are Hodrick (1992) two-sided p -values (denoted as H, p -val), and two-sided p -values from the parametric bootstrap (denoted as PB, p -val). Adjusted R^2 is reported as \bar{R}^2 , and the Durbin-Watson statistic is shown as $[DW]$. The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. All variables are measured in monthly units. The sample period is 09/1998 to 09/2008 (121 observations).

Dependent variable		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	\bar{R}^2 [DW]	Joint p -val
Conference Board coincident index	Coef.	0.000	-0.403	-0.111	0.381	0.198	5.57%	
	NW-7, p -val	0.576	0.010	0.455	0.000	0.055	[1.75]	0.001
	H, p -val	0.571	0.018	0.560	0.011	0.022		0.017
	PB, p -val	0.545	0.044	0.613	0.013	0.091		0.076
NAPM	Coef.	0.010	-8.220	4.343	-0.995	2.211	9.92%	
	NW-2, p -val	0.223	0.000	0.197	0.569	0.136	[1.99]	0.001
	H, p -val	0.211	0.016	0.197	0.604	0.204		0.056
	PB, p -val	0.368	0.007	0.236	0.637	0.336		0.069
Capacity utilization	Coef.	0.003	-1.272	0.286	0.055	0.242	7.15%	
	NW-7, p -val	0.129	0.000	0.431	0.846	0.103	[1.51]	0.000
	H, p -val	0.110	0.003	0.480	0.854	0.288		0.008
	PB, p -val	0.245	0.006	0.463	0.880	0.186		0.029
Civilian unemployment	Coef.	0.001	5.345	-1.019	-1.642	-1.971	4.51%	
	NW-2, p -val	0.876	0.004	0.554	0.191	0.037	[2.24]	0.037
	H, p -val	0.883	0.025	0.624	0.205	0.065		0.154
	PB, p -val	0.890	0.058	0.620	0.343	0.131		0.264
Retail sales	Coef.	0.001	-0.493	-1.146	1.179	0.782	1.30%	
	NW-19, p -val	0.552	0.616	0.311	0.004	0.001	[2.68]	0.000
	H, p -val	0.624	0.726	0.319	0.053	0.193		0.372
	PB, p -val	0.637	0.626	0.260	0.043	0.130		0.325
Housing starts	Coef.	-0.005	-3.934	-1.899	2.977	2.484	-1.98%	
	NW-6, p -val	0.679	0.308	0.610	0.199	0.261	[2.73]	0.584
	H, p -val	0.789	0.301	0.625	0.329	0.242		0.704
	PB, p -val	0.656	0.577	0.661	0.233	0.305		0.759

structure variables as predictors. At the same time, estimation results (not reported) show that $f_t^{(3)}$ and $f_t^{(4)}$ are insignificant in this case, and have little marginal explanatory power, hence, we opt to maintain a frugal set of predictors.

Several empirical observations are in order, based on Table 4. A rise in $y_t^{(1)}$ induces an increase, while a rise in $f_t^{(2)}$ induces a decline in Treasury bill returns, and this effect is statistically significant. At a closer look, this finding contrasts with the results in Table 2 related to real economic activity. The reversal in the signs of the slope coefficients is consistent with the intuition that Treasury bill returns rise when real economic activity falls, and vice versa, and thus indicates that the connection between Treasury bill returns and real economic activity can be revealed through forward variances.

The Hodrick p -values on $y_t^{(1)}$ and $f_t^{(2)}$ indicate that both front-end forward variances are significant predictors of future excess Treasury returns. It also follows that a change of one standard deviation in $y_t^{(1)}$ can lead to as much as a 14 basis point increase in Treasury bill returns.

When predicting excess returns of Treasury bills, with or without the forward variances, $credit_t$ and $yslope_t$ have positive and negative coefficients, respectively, and are individually significant. Note that when $credit_t$ and $yslope_t$ alone are used as predictors, the Hodrick p -value

for joint parameter insignificance is 0.094, while the counterpart p -value with the two forward variances alone is 0.010. This finding strengthens the evidence for predictive power of forward variances, which has already been demonstrated by the results in Tables 2 and 3, related to real economic activity. An adjusted R^2 of 12.27% obtains when the two front-end forward variances are combined with the credit spread and the slope of the yield curve. The Hodrick joint p -value of 0.004 for the specification that includes all four predictors provides further substantiation of statistical significance.

In contrast, performing similar analysis on Treasury bond returns reveals no evidence of predictability. First, the slope coefficients are generally insignificant. Second, the p -values for the joint χ^2 test in the last column, none lower than 0.152, broadly confirm the lack of predictability.

We further find that our main results on the importance of $y_t^{(1)}$ and $f_t^{(2)}$, as reported in Table 4, still hold after incorporating a set of additional predictors, as evidenced in Table A2. While the adjusted R^2 worsens when $rdef_t$ (the return difference between Moody's Baa bond and the ten-year Treasury bond) replaces $credit_t$, the predictability with forward variances remains intact, with the joint p -value of 0.004. A qualitatively similar picture about the contribution of $y_t^{(1)}$ and $f_t^{(2)}$ emerges when $rdef_t$, or $rterm_t$

Table 4

Predicting returns of Treasuries with forward variances.

This table shows the results from regressions of returns of Treasury bills and bonds on prior month forward variances and other predictors. r_t^{bills} are total returns of Treasuries with maturity less than one year, and r_t^{bonds} are total returns of Treasuries with maturity greater than ten years (data source: Morningstar, previously Ibbotson Associates). All variables are measured in monthly units. Reported are coefficient estimates and two-sided p -values for the following regressions over 09/1998 to 09/2008 (121 observations):

$$\ln(1 + r_{t+1}^{\text{treas market}}) - \ln(1 + r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 \text{credit}_t + \theta_2 \text{yslope}_t + \varepsilon_{t+1},$$

where r_t^* is the one-month riskfree rate, and credit_t denotes the credit spread, defined as Moody's Baa bond yield minus Aaa bond yield. yslope_t denotes the slope of the Treasury yield curve, defined as the ten-year Treasury yield minus the three-month Treasury yield (data source: Federal Reserve Board). To correct for autocorrelation and heteroskedasticity, we use the Newey-West estimator with automatically selected lag, as in Newey and West (1994), and the reported two-sided p -values are denoted as NW-lag, p -val. Shown also are Hodrick (1992) two-sided p -values (denoted as H, p -val), and two-sided p -values from the parametric bootstrap (denoted as PB, p -val). Adjusted R^2 is reported as \bar{R}^2 , and the Durbin-Watson statistic is shown as $\lfloor DW \rfloor$. The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. Robustness to alternative choices of the predictors, additional to forward variances, is established in Table A2.

Dependent variable		Const.	$y_t^{(1)}$	$f_t^{(2)}$	credit_t	yslope_t	\bar{R}^2 $\lfloor DW \rfloor$	Joint p -val
Bills	Coef.	0.000	0.043	-0.051			2.30%	
	NW-3, p -val	0.205	0.018	0.001			$\lfloor 1.72 \rfloor$	0.002
	H, p -val	0.194	0.015	0.003				0.010
	PB, p -val	0.325	0.071	0.018				0.070
Bills	Coef.	-0.000			0.693	-0.050	7.74%	
	NW-5, p -val	0.015			0.017	0.099	$\lfloor 1.93 \rfloor$	0.056
	H, p -val	0.029			0.035	0.063		0.094
	PB, p -val	0.011			0.007	0.141		0.013
Bills	Coef.	-0.000	0.046	-0.064	0.769	-0.078	12.27%	
	NW-6, p -val	0.042	0.015	0.000	0.008	0.031	$\lfloor 1.94 \rfloor$	0.000
	H, p -val	0.052	0.008	0.000	0.024	0.020		0.004
	PB, p -val	0.079	0.034	0.005	0.003	0.042		0.004
Bonds	Coef.	0.008	1.306	-2.179			0.07%	
	NW-2, p -val	0.162	0.399	0.156			$\lfloor 2.03 \rfloor$	0.306
	H, p -val	0.247	0.388	0.156				0.349
	PB, p -val	0.207	0.439	0.176				0.372
Bonds	Coef.	0.005			-9.393	4.027	0.61%	
	NW-3, p -val	0.579			0.459	0.053	$\lfloor 2.05 \rfloor$	0.152
	H, p -val	0.619			0.505	0.079		0.212
	PB, p -val	0.662			0.524	0.116		0.299
Bonds	Coef.	0.006	1.448	-1.669	-7.701	3.552	-0.21%	
	NW-2, p -val	0.570	0.347	0.318	0.587	0.172	$\lfloor 2.05 \rfloor$	0.266
	H, p -val	0.610	0.348	0.303	0.597	0.197		0.362
	PB, p -val	0.610	0.384	0.328	0.593	0.211		0.479

(the return difference between the ten-year Treasury bond and the three-month Treasury bill), replace yslope_t .

The gist of our analysis is that front-end forward variances are useful in tracking future movements in Treasury bill returns, but fail to capture the corresponding movements in Treasury bond returns.

4.4. Forward variances do not have marginal predictive content for Treasury and corporate bond returns in the presence of the Cochrane and Piazzesi (2005) factor

How should one interpret the finding that forward variances pick up variation in short-duration assets like Treasury bills, but not in long-duration assets like Treasury bonds? We pursue here two different approaches.

First, we add to the predictive regression (11) the Cochrane and Piazzesi (2005) factor (denoted CP_t), which has been shown to capture a large fraction of the variation in the excess returns of bonds with maturity between two and five years (see also, among others, Ludvigson and Ng, 2009). Panels A and B of Table 5 uncover that CP_t is highly

significant when predicting Treasury bond returns, with a maximum p -value of 0.012, and leads to an increase in the adjusted R^2 (compare Tables 4 and 5), but, but the forward variances remain insignificant. However, when predicting Treasury bill returns, the coefficients on CP_t have a minimum Hodrick p -value of 0.155, while those on $y_t^{(1)}$ and $f_t^{(2)}$ are still significant, as in Table 4.

Second, we consider another long-duration asset and examine the predictability of excess returns of corporate bonds (from Morningstar). Panel C of Table 5 shows again that forward variances remain insignificant predictors in the presence of CP_t . Overall, our findings lend some credence to the contention that forward variances can be useful for tracking short-run business conditions.

4.5. Evidence favors a link between forward variances and subsequent stock market returns

Elaborating further on the role of forward variances in explaining asset returns, we focus on stock market returns

Table 5

Predicting returns of Treasury and corporate bonds, adding the Cochrane and Piazzesi (2005) factor.

This table shows the results from regressions of returns of Treasury bills and bonds, and corporate bonds, on prior month forward variances, the Cochrane and Piazzesi (2005) factor (denoted CP_t), and other predictors. r_t^{bills} are total returns of Treasuries with maturity less than one year, r_t^{bonds} are total returns of Treasuries with maturity greater than ten years, and r_t^{corp} are total returns of corporate bonds (data source: Morningstar, previously Ibbotson Associates) with average maturity of 20 years. All variables are measured in monthly units. Reported are coefficient estimates and two-sided p -values. r_t^* is the one-month riskfree rate, and $credit_t$ denotes the credit spread, defined as Moody's Baa bond yield minus Aaa bond yield. $yslope_t$ denotes the slope of the Treasury yield curve, defined as the ten-year Treasury yield minus the three-month Treasury yield (data source: Federal Reserve Board). The Cochrane and Piazzesi factor over our sample is extracted from CRSP, using Matlab code provided by Monika Piazzesi on her Web site. To correct for autocorrelation and heteroskedasticity, we use the Newey-West estimator with automatically selected lag, as in Newey and West (1994), and the reported two-sided p -values are denoted as NW-lag, p -val. Shown also are Hodrick (1992) two-sided p -values (denoted as H, p -val), and two-sided p -values from the parametric bootstrap (denoted as PB, p -val). Adjusted R^2 is reported as \bar{R}^2 , and the Durbin-Watson statistic is shown as $[DW]$. The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. The sample period is 09/1998 to 09/2008 (121 observations).

Variable		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$credit_t$	$yslope_t$	CP_t	\bar{R}^2 [DW]	Joint p -val
<i>Panel A: $\ln(1+r_{t+1}^{bills})-\ln(1+r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 credit_t + \theta_2 yslope_t + \theta_3 CP_t + \varepsilon_{t+1}$</i>									
Bills	Coef.	0.000	0.050	-0.050		0.001	-0.037	1.53%	
	NW-2, p -val	0.178	0.005	0.001		0.965	0.208	[1.77]	0.001
	H, p -val	0.257	0.004	0.005		0.969	0.223		0.007
	PB, p -val	0.442	0.014	0.008		0.970	0.587		0.058
Bills	Coef.	-0.000	0.055	-0.052	0.576		-0.032	9.63%	
	NW-0, p -val	0.025	0.001	0.000	0.021		0.137	[1.91]	0.001
	H, p -val	0.044	0.001	0.002	0.039		0.155		0.005
	PB, p -val	0.194	0.013	0.009	0.089		0.518		0.066
<i>Panel B: $\ln(1+r_{t+1}^{bonds})-\ln(1+r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 credit_t + \theta_2 yslope_t + \theta_3 CP_t + \varepsilon_{t+1}$</i>									
Bonds	Coef.	0.005	-0.078	-2.286		0.097	7.476	6.29%	
	NW-3, p -val	0.508	0.961	0.119		0.967	0.000	[1.90]	0.000
	H, p -val	0.580	0.962	0.154		0.970	0.005		0.049
	PB, p -val	0.580	0.967	0.170		0.971	0.010		0.011
Bonds	Coef.	0.004	-0.069	-2.309	2.401		7.533	6.32%	
	NW-3, p -val	0.775	0.965	0.111	0.855		0.000	[1.90]	0.000
	H, p -val	0.775	0.965	0.137	0.860		0.004		0.045
	PB, p -val	0.835	0.970	0.172	0.884		0.012		0.020
<i>Panel C: $\ln(1+r_{t+1}^{corp})-\ln(1+r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 credit_t + \theta_2 yslope_t + \theta_3 CP_t + \varepsilon_{t+1}$</i>									
Corporate	Coef.	-0.001	-0.605	-0.865		0.913	6.408	4.70%	
	NW-3, p -val	0.919	0.608	0.441		0.667	0.001	[1.77]	0.001
	H, p -val	0.927	0.660	0.512		0.680	0.008		0.068
	PB, p -val	0.911	0.630	0.433		0.965	0.004		0.071
Corporate	Coef.	0.002	-0.736	-0.992	-0.949		6.749	4.80%	
	NW-3, p -val	0.874	0.542	0.359	0.948		0.000	[1.76]	0.000
	H, p -val	0.872	0.588	0.439	0.951		0.003		0.071
	PB, p -val	0.927	0.703	0.500	0.701		0.010		0.038

and consider the forecasting relation

$$\ln(1+r_{t+1}^{stock\ market})-\ln(1+r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 yslope_t + \theta_2 (e/p)_t + \varepsilon_{t+1}. \quad (12)$$

Stock market returns $r_{t+1}^{stock\ market}$ in (12) are measured by the total monthly returns of both the S&P 500 index and the MSCI World Capitalization index (from Datastream). We reiterate that our fundamental interest is in assessing the predictable variation captured by the model using the χ^2 test statistic, based on standard asymptotic inference, as well as a parametric bootstrap procedure and the associated joint p -values.

Guided by the preceding evidence, we combine the two front-end forward variances with one predictor, related to the term structure, and with a valuation ratio. The earnings yield on the S&P 500 index (from Global Financial Data, and denoted by $(e/p)_t$), is employed as the

valuation ratio (e.g., Lewellen, 2004). The term structure variable here is the slope of the yield curve $yslope_t$, although, as before, we also examine predictive regressions that incorporate $credit_t$, $rdef_t$, and $rterm_t$, to assess robustness. Eq. (12) lets us investigate whether the null hypothesis of zero slope coefficients on our set of forward variances can be rejected in the presence of a host of additional stock market predictors.

In the full specification (12), the coefficient on $f_t^{(2)}$ is significant for the S&P 500, with p -values of 0.000, 0.005, and 0.001, based on the inference methods of Newey and West, Hodrick, and the parametric bootstrap, respectively (see Table 6). The results with MSCI reveal a consistent picture, with p -values of 0.000, 0.004, and 0.000, for the coefficient on $f_t^{(2)}$, respectively.

Summarizing, a higher $f_t^{(2)}$ positively impacts future stock returns and real economic activity but adversely affects future excess Treasury bill returns, and the effect is

Table 6

Predicting excess returns in the stock market.

This table shows the results from regressions of excess stock market returns on prior month forward variances and economic fundamentals. For robustness, we use both the S&P 500 index and the MSCI World Capitalization index (data source: Datastream). Reported are coefficient estimates and two-sided p -values for the following regressions over the sample period 09/1998 to 09/2008 (121 observations):

$$\ln(1 + r_{t+1}^{\text{stock market}}) - \ln(1 + r_t^*) = \alpha + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 \text{yslope}_t + \theta_2 (e/p)_t + \varepsilon_{t+1},$$

where r_t^* is the one-month riskfree rate, and $(e/p)_t$ is the earnings-to-price ratio (data source: Global Financial Data). Thus, we combine here a term structure variable and a cash flow variable with the first two forward variances. yslope_t denotes the slope of the Treasury yield curve, defined as the ten-year Treasury yield minus the three-month Treasury yield (data source: Federal Reserve Board). To correct for autocorrelation and heteroskedasticity, we use the Newey–West estimator with automatically selected lag, as in Newey and West (1994), and the reported two-sided p -values are denoted as NW-lag, p -val. Shown also are Hodrick (1992) two-sided p -values (denoted as H, p -val), and two-sided p -values from the parametric bootstrap (denoted as PB, p -val). Adjusted R^2 is reported as \bar{R}^2 , and the Durbin–Watson statistic is shown as $[DW]$. The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. Robustness to alternative choices of the predictors, additional to forward variances, is established in Table A3. All variables are measured in monthly units.

Dependent variable		Const.	$y_t^{(1)}$	$f_t^{(2)}$	yslope_t	$(e/p)_t$	\bar{R}^2 [DW]	Joint p -val
S&P 500	Coef.	−0.012	−3.842	5.467			2.24%	
	NW-3, p -val	0.209	0.177	0.084			[1.79]	0.203
	H, p -val	0.222	0.194	0.071				0.185
	PB, p -val	0.239	0.151	0.037				0.111
S&P 500	Coef.	−0.032			1.160	8.844	1.99%	
	NW-2, p -val	0.092			0.760	0.033	[1.81]	0.075
	H, p -val	0.089			0.757	0.031		0.074
	PB, p -val	0.060			0.783	0.044		0.138
S&P 500	Coef.	−0.165	−0.770	9.915	15.971	25.390	14.64%	
	NW-1, p -val	0.000	0.796	0.000	0.009	0.000	[1.70]	0.000
	H, p -val	0.003	0.814	0.005	0.027	0.001		0.008
	PB, p -val	0.000	0.778	0.001	0.005	0.000		0.001
MSCI	Coef.	−0.004	−6.564	6.801			4.35%	
	NW-0, p -val	0.674	0.033	0.014			[1.62]	0.049
	H, p -val	0.707	0.042	0.036				0.099
	PB, p -val	0.695	0.017	0.008				0.029
MSCI	Coef.	−0.039			2.213	10.673	3.33%	
	NW-1, p -val	0.042			0.569	0.009	[1.67]	0.023
	H, p -val	0.037			0.556	0.008		0.024
	PB, p -val	0.031			0.608	0.016		0.065
MSCI	Coef.	−0.144	−3.759	10.822	13.982	23.656	14.60%	
	NW-2, p -val	0.000	0.254	0.000	0.015	0.000	[1.59]	0.000
	H, p -val	0.006	0.275	0.004	0.040	0.001		0.004
	PB, p -val	0.001	0.178	0.000	0.016	0.000		0.001

generally significant for the S&P 500, MSCI, and Treasury bill returns. The estimation results imply that a change of one standard deviation in $f_t^{(2)}$ can result in about 1.34% (1.63%) change in monthly excess returns on the S&P 500 (MSCI) index, *ceteris paribus*.

Overall, the significance of the slope coefficients on $y_t^{(1)}$ and $f_t^{(2)}$, reported in Table 6, together with our observations from Tables 2 and 4, are indicative of a broad economic framework, where front-end forward variances can help predict real economic activity, Treasury bill returns, and stock market returns. For instance, while a higher value of $y_t^{(1)}$ is associated with a subsequent fall in excess stock returns and in real economic activity, it tends to increase Treasury bill returns (see Table 4). Based on the estimated p -values, the effect is statistically significant at conventional levels for Treasury bills and both indicators of real activity, and marginally significant for stock index returns.

As in previously reported predictive regressions, the coefficient on the earnings yield is positive, implying that expected returns are small when $(e/p)_t$ is small (e.g., for recent evidence, see Goyal and Welch, 2008). Furthermore,

this coefficient is significant, according to all three methods of inference. In our sample, yslope_t is significant in the full specification.

What we also learn from this exercise is that the null hypothesis of joint parameter insignificance is rejected for the specification (12), with p -values below 0.008 for each of the three methods of inference. Thus, we reject the null hypothesis that the forward variances, in addition to the yield curve slope and the earnings yield, are redundant in explaining stock returns. The message from the χ^2 test is that there appears to be a link between forward variances and future stock market returns, controlling for other predictors.

Using $(e/p)_t$ and yslope_t , in the absence of forward variances, yields an adjusted R^2 of 1.99% (3.33%) for the S&P 500 (MSCI) index, while the two forward variances alone generate an adjusted R^2 of 2.24% (4.35%). The joint explanatory power of the four predictors together, as measured by the adjusted R^2 , rises to 14.64% (14.60%).

Table A3 provides further evidence that $f_t^{(2)}$ is a reliable predictor of stock market returns in the presence of alternative term structure predictors. We observe that

Table 7

Predicting excess stock market returns at three- and six-month horizons with overlapping observations.

This table shows the results from longer-horizon predictive regressions of excess stock returns on forward variances and economic fundamentals. For robustness, we use both the S&P 500 index and the MSCI World Capitalization index (data source: Datastream). Reported are coefficient estimates and two-sided *p*-values for the following regressions over the sample period 09/1998 to 09/2008:

$$er_{t+j}^{\text{stock market}} = \beta_0 + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \theta_1 \text{yslope}_t + \theta_2 (e/p)_t + \varepsilon_{t+j},$$

where $er_{t+j}^{\text{stock market}}$, for $j=3$ ($j=6$), denotes cumulative excess log returns over the three (six) months following month t , respectively. yslope_t denotes the slope of the Treasury yield curve, defined as the ten-year Treasury yield minus the three-month Treasury yield, and $(e/p)_t$ is the earnings-to-price ratio (data source: Federal Reserve Board and Global Financial Data). Shown also are the Hodrick (1992) two-sided *p*-values (denoted H, *p*-val) for the coefficient estimates. Adjusted R^2 is reported as \bar{R}^2 . The final column reports the Hodrick (1992) *p*-value for the null hypothesis that the slope coefficients are jointly equal to zero.

Horizon		Const.	$y_t^{(1)}$	$f_t^{(2)}$	yslope_t	$(e/p)_t$	\bar{R}^2	Joint <i>p</i> -val	
S&P 500									
3 Months	Coef.	-0.384	5.398	15.564	38.649	60.327	25.38%	0.006	
	H, <i>p</i> -val	0.001	0.364	0.001	0.032	0.001			
6 Months	Coef.	-0.479	0.914	21.304	53.387	76.174	22.82%		
	H, <i>p</i> -val	0.007	0.895	0.002	0.059	0.008			
MSCI									
3 Months	Coef.	-0.344	0.930	15.364	35.913	58.425	21.20%		0.003
	H, <i>p</i> -val	0.002	0.875	0.001	0.039	0.001			
6 Months	Coef.	-0.430	-5.245	21.101	52.280	78.690	22.24%		
	H, <i>p</i> -val	0.012	0.430	0.002	0.058	0.007			

the *p*-values for the Hodrick test of joint parameter insignificance are all below 0.041.⁸

One question of interest remains: Is the predictive ability of forward variances maintained at different return horizons? This question seems relevant in light of the ongoing debate whether predictability extends beyond the monthly horizon (e.g., Fama and French, 1988; Cutler, Poterba, and Summers, 1991; Hodrick, 1992; Campbell, 2001; Ang and Bekaert, 2007; Boudoukh, Richardson, and Whitelaw, 2008). Given the length of our time series of forward variances, we present in Table 7 results for stock market returns at three- and six-month forecast horizons, with overlapping observations. Note that, as in the regressions with one-month forecasting horizon, the slope coefficient associated with $f_t^{(2)}$ is positive, remains significant in the presence of both yslope_t and $(e/p)_t$, and has Hodrick *p*-values below 0.05. Equally robust is the rejection of joint parameter insignificance, with Hodrick *p*-values below 0.05. Consistent with Boudoukh, Richardson, and Whitelaw (2008), an enhanced degree of predictability over longer than monthly horizons is implied by the higher values of adjusted R^2 .

Can we statistically reject the hypothesis of joint predictability across one-, three-, and six-month horizons for stock market returns? Stacking up the corresponding equations, and using the same set of predictors as in Table 7, we test for zero slope coefficients across the three horizons, as in Ang and Bekaert (2007, Appendix B, Eq. (B7)), but adopt the covariance matrix estimator

⁸ To further examine the robustness of the predictive relations in Table 6, we have performed analogous regressions where the variance risk premium (following Bakshi and Madan, 2006; Carr and Wu, 2009; Bollerslev, Tauchen, and Zhou, 2009) is included as a predictor instead of yslope_t . While the Hodrick *p*-values (not reported) show that $f_t^{(2)}$ preserves its significance, the adjusted R^2 's vary.

of Hodrick (1992). The χ^2 test statistics and the corresponding *p*-values are as follows,

	χ^2	df	Joint <i>p</i> -val
S&P 500	37.96	12	0.0002
MSCI	49.53	12	0.0000

Together with our previous findings, these test results present a coherent picture of predictability at the one-, three-, and six-month horizons for stock market returns.

What is a plausible interpretation of the overall results documented so far? First, we reiterate that our findings show that forward variances can predict excess returns of Treasury bills, but not of Treasury bonds. This suggests that forward variances may reflect business conditions in the short run, but not in the long run. Second, we observe a link between forward variances and economic activity, as well as excess returns of stock market indexes. In summary, the variation in forward variances extracted from the option market appears to be related to short-run future movements in both the financial and the real sector.

4.6. Simulation evidence that the Hodrick (1992) test offers the correct size, even with nearly integrated predictors

In light of our previous discussion, two fundamental concerns remain to be addressed. First, how might the predictive regression inference be affected when some of the predictors follow ARMA(1,1)-GARCH(1,1) dynamics, and, in particular, if they are allowed to be nearly integrated? Second, what is the potential impact of a small sample size in this context?

We investigate these concerns within a Monte Carlo simulation experiment, and focus on the empirical size of the Hodrick (1992) test for joint parameter insignificance

Table 8

Empirical size of the Hodrick (1992) joint test, with nearly integrated ARMA(1,1) predictors.

Denote by $C_{5\%}$ and $C_{1\%}$ the χ^2 cut-off values for the Hodrick (1992) test of joint parameter insignificance, at nominal size 5% and 1%, respectively. Adopting a setup with four predictors, the empirical sizes of the joint test is computed as follows. Step 1: Estimate an ARMA(1,1)-GARCH(1,1) model on each predictor from the regressions, reported in Tables 2, 4, 6 and 10 which, which involve four predictors. Store the standardized residuals from these estimations, together with the residuals from the respective predictive regressions. Step 2: Consider ARMA(1,1)-GARCH(1,1) parameter sets, where some AR coefficients are close to unity, and with negative MA components. We fix the ARCH parameter to 0.25 and the GARCH parameter to 0.74. Step 3: For each parameter set, build iteratively time series of bootstrapped predictors with 121 observations, using the standardized residuals from Step 1. The bootstrap is performed 50,000 times. Step 4: Bootstrap also the predicted variable, under the null of no predictability. In Steps 3 and 4, the residuals are bootstrapped in a pairwise manner, to preserve the dependencies among them. Step 5: For each set of bootstrapped predicted variable and four bootstrapped predictors, calculate the χ^2 statistic for the Hodrick (1992) test of joint parameter insignificance. The reported empirical sizes are the proportion of these χ^2 statistics, which exceed $C_{5\%}$ or $C_{1\%}$, respectively. The log changes in variance swap rates of different maturities (as defined in Eq. (14)) are indexed by $n=1,2,3,4$, and denoted by $q_{t+1}^{\text{var swap}(n)}$.

	Parameter Set 1		Parameter Set 2		Parameter Set 3		Parameter Set 4	
	AR(1)	MA(1)	AR(1)	MA(1)	AR(1)	MA(1)	AR(1)	MA(1)
Predictor 1	0.95	-0.20	0.99	-0.20	0.95	-0.20	0.99	-0.20
Predictor 2	0.95	-0.20	0.99	-0.20	0.95	-0.20	0.99	-0.20
Predictor 3	0.99	-0.40	0.99	-0.40	0.99	-0.60	0.99	-0.60
Predictor 4	0.99	-0.40	0.99	-0.40	0.99	-0.60	0.99	-0.60
Nominal size	5%	1%	5%	1%	5%	1%	5%	1%
Residuals from:								
Non-farm payroll	0.087	0.016	0.086	0.014	0.057	0.009	0.054	0.009
Industrial production	0.079	0.012	0.089	0.015	0.052	0.007	0.060	0.008
Treasury bills	0.039	0.004	0.037	0.003	0.038	0.004	0.035	0.004
S&P 500	0.044	0.005	0.044	0.005	0.053	0.008	0.049	0.008
MSCI	0.053	0.010	0.054	0.009	0.055	0.010	0.054	0.009
$q_{t+1}^{\text{var swap}(1)}$	0.053	0.006	0.054	0.007	0.064	0.008	0.066	0.009
$q_{t+1}^{\text{var swap}(2)}$	0.052	0.008	0.054	0.007	0.060	0.009	0.061	0.009
$q_{t+1}^{\text{var swap}(3)}$	0.055	0.009	0.049	0.008	0.058	0.010	0.054	0.008
$q_{t+1}^{\text{var swap}(4)}$	0.037	0.004	0.035	0.003	0.036	0.004	0.032	0.003

(based on the χ^2 statistic). Ang and Bekaert (2007) and Wei and Wright (2009), among others, have found this test to exhibit correct size in small samples, which has partly motivated us to adopt the test to study predictability in our setting. Adhering to convention, we consider 5% and 1% nominal sizes of the test.

To accommodate the issue of possibly nearly integrated predictors, we include parameter sets with autoregressive parameters close to unity, and with sizable negative MA component. Simulation results are reported in Table 8 for the case with four predictors, when the predicted variable is simulated under the null of no predictability (see, e.g., Amihud, Hurvich, and Wang, 2009, Appendix 3; and Cochrane, 2008).

The simulated time series for the predictors are built iteratively, following the ARMA(1,1)-GARCH(1,1) specification, and using bootstrapped residuals from actual predictive regressions, as suggested, for example, in Mark (1995). This approach ensures that our conclusions reflect more closely the properties of the actual data. Moreover, all bootstrapped series match the length of our sample, hence, the findings pertain to the small sample issue. Finally, the bootstrap is performed in a fashion that preserves the dependencies across residuals.

Table 8 is informative from two angles. First, the Hodrick test of joint parameter insignificance exhibits empirical sizes in the neighborhood of the respective nominal ones. Even for parameter set 4, with all AR(1) parameters equal to 0.99, and

some sizable negative MA(1) parameters, the size distortion is limited, with empirical sizes ranging between 0.032 and 0.066 for the 5% nominal size, and between 0.003 and 0.009 for the 1% nominal size. Second, we find no evidence that the test consistently over-rejects the null hypothesis of no predictability. In fact, for more than half of the reported configurations, the test appears to be somewhat conservative, in particular for the 1% nominal size. Therefore, the simulation exercise affirms our conclusions about the predictability of asset returns and real economic activity.

4.7. Bootstrap evidence conveys similar message, while accounting for small sample size

In line with our efforts to assess statistical robustness, we have performed a parametric bootstrap, as in Amihud, Hurvich, and Wang (2009) and Mark (1995). Such a procedure accounts for the effect of small sample size on predictive inference, as emphasized also in Section 4.1. To make the bootstrap realistic, we accommodate persistence in each regressor and its conditional variance.

The bootstrap exercise establishes a number of points. At the outset, Tables 2–6 show that our results on individual and joint p -values cohere well with those based on the asymptotic standard errors. For example, both the parametric bootstrap and the Hodrick p -values do not reject the lack of predictability of Treasury bond returns. Probing further, our results reveal that the number of significant joint p -values

based on the parametric bootstrap are comparable to those from the Hodrick test for joint insignificance. At the same time, there seems to be no discernible pattern in the p -values for the individual slope coefficients, which brings some deviation from a complete agreement. It does appear that whenever the individual Hodrick p -value is significant, so is the corresponding parametric bootstrap p -value.

In sum, when the parametric bootstrap is done to account for possible persistence in the first and the second moments of the regressors, the results can be viewed as providing yet another dimension of robustness. The predictability is statistically present even when inference is based on the parametric bootstrap.

4.8. Joint predictability across markets reinforces the evidence

To reinforce our findings from single regressions, we examine the joint predictability across the Treasury market, the stock market, and real economic activity by considering a system of equations of the type

$$\mathbf{z}_{t+1} = \mathbf{a} + \mathbf{B}\mathbf{f}_t + \mathbf{e}_{t+1}, \tag{13}$$

where \mathbf{B} is an $\mathbb{L} \times \mathbb{K}$ matrix of regression slope coefficients, \mathbb{L} is the number of equations, and \mathbb{K} denotes the number of predictors. The vector of dependent variables in (13) can be, for instance, $\mathbf{z}_{t+1} \equiv [g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}]$,

where er_{t+1} denotes excess returns, in which case $\mathbb{L} = 3$. We examine whether our conclusions about predictability are affected if one considers a system of regression equations across markets, as opposed to single regressions.

Using the Hodrick (1992) procedure, we test whether all slope coefficients are zero, i.e., $\mathbf{B} = \mathbf{0}_{\mathbb{L} \times \mathbb{K}}$. The importance of testing for joint significance in the context of multiple predictive regressions with common predictors has been emphasized by Boudoukh, Richardson, and Whitelaw (2008) and Cochrane (2008).

Table 9 separately shows results with $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)} f_t^{(3)}]'$ and $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)}]'$. As seen, for example, from row 7 in Panel A, when $\mathbf{z}_{t+1} \equiv [g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}]$, we obtain a chi-squared statistic $\chi^2(9) = 20.3$, with an associated p -value of 0.016. Hence, we reject the null hypothesis of joint parameter insignificance with $[y_t^{(1)} f_t^{(2)} f_t^{(3)}]'$. At the same time, Panel B shows that with $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)}]'$, the p -values are higher, suggesting that augmenting the vector of forward variances with $f_t^{(3)}$ enhances predictability across markets.

Similar conclusions can be inferred from other combinations for the vector of dependent variables. Specifically, there are 17 p -values below 0.05 and 25 p -values below 0.10, out of the 30 joint p -values reported in Table 9. Putting it all together, the main message remains that forward variances help predict growth in real economic

Table 9
Exclusion test for joint predictability based on a system of equations.

All results reported in this table are based on the Hodrick (1992) covariance matrix estimator, under the null hypothesis of no predictability. We consider a system of equations:

$$\mathbf{z}_{t+1} = \mathbf{a} + \mathbf{B}\mathbf{f}_t + \mathbf{e}_{t+1},$$

where \mathbf{z}_{t+1} is a vector of dependent variables, \mathbf{B} is a matrix of regression slope coefficients, \mathbf{f}_t is a vector of forward variances, and \mathbf{e}_{t+1} is a vector of regression residuals. We show separately results for $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)} f_t^{(3)}]'$, and $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)}]'$, respectively. Reported are the χ^2 test statistics, the degrees of freedom, and the p -values for the Hodrick (1992) test that the slope coefficients are jointly equal to zero. The vectors of dependent variables include combinations of (i) growth in real economic activity, (ii) excess returns in the stock market and of Treasury bills (e.g., denoted by $er_{t+1}^{\text{S\&P 500}}$), and (iii) log changes in variance swap rates of different maturities (indexed by $n=1,2,3,4$, and denoted by $q_{t+1}^{\text{var swap}(n)}$).

No.	Vector of dependent variables	Panel A. $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)} f_t^{(3)}]'$			Panel B. $\mathbf{f}_t \equiv [y_t^{(1)} f_t^{(2)}]'$		
		χ^2	df	Joint p -val	χ^2	df	Joint p -val
1	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{S\&P 500}}]$	11.5	6	0.075	7.3	4	0.120
2	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{MSCI}}]$	12.6	6	0.050	8.4	4	0.078
3	$[er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}]$	13.3	6	0.038	10.4	4	0.034
4	$[er_{t+1}^{\text{bills}}, er_{t+1}^{\text{MSCI}}]$	15.5	6	0.017	12.6	4	0.014
5	$[g_{t+1}^{\text{indus prod}}, er_{t+1}^{\text{S\&P 500}}]$	8.8	6	0.183	8.2	4	0.084
6	$[g_{t+1}^{\text{indus prod}}, er_{t+1}^{\text{MSCI}}]$	9.7	6	0.140	9.2	4	0.057
7	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}]$	20.3	9	0.016	12.3	6	0.056
8	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{MSCI}}]$	23.0	9	0.006	14.5	6	0.024
9	$[g_{t+1}^{\text{indus prod}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}]$	16.0	9	0.068	13.6	6	0.035
10	$[g_{t+1}^{\text{indus prod}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{MSCI}}]$	18.0	9	0.035	15.7	6	0.015
11	$[q_{t+1}^{\text{var swap}(1)}, q_{t+1}^{\text{var swap}(2)}, q_{t+1}^{\text{var swap}(3)}, q_{t+1}^{\text{var swap}(4)}]$	29.6	12	0.003	6.4	8	0.601
12	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}, q_{t+1}^{\text{var swap}(1)}]$	24.6	12	0.017	15.2	8	0.056
13	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}, q_{t+1}^{\text{var swap}(2)}]$	26.7	12	0.008	17.1	8	0.029
14	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}, q_{t+1}^{\text{var swap}(3)}]$	31.3	12	0.002	13.0	8	0.110
15	$[g_{t+1}^{\text{payroll}}, er_{t+1}^{\text{bills}}, er_{t+1}^{\text{S\&P 500}}, q_{t+1}^{\text{var swap}(4)}]$	22.3	12	0.035	14.4	8	0.072

activity and excess asset returns, both within single regressions and a system of regressions.

A possible unifying conclusion is that the reported empirical results validate the presence of a predictable component in excess asset returns and the growth of real economic activity, consistent with some of the evidence from other financial markets (e.g., see, among others, Bekaert and Hodrick, 1992; Campbell and Shiller, 1988; Cutler, Poterba, and Summers, 1991; and Fama and French, 1993; Mark, 1995). Confirming predictability, the joint parameter insignificance is mostly rejected across markets in our sample. What appears to be new here is that part of the predictability is attributable to a set of forward variances.

4.9. Changes in variance swap rates can be predicted through forward variances

Drawing on the themes of the paper, we pose the final question: Do forward variances track changes in variance swap rates? Our aim here is to understand the behavior of traded market volatility, in the time series and over different horizons, by examining the price dynamics of assets sensitive to movements in integrated variance.

A variance swap is a zero-cost instrument at entry and, hence, we take the dependent variable to be the log change in a variance swap rate. Maintaining the set of regressors based on forward variances, we obtain the

regression

$$q_{t+1}^{var\ swap(n)} \equiv \ln\left(\frac{VS_{t+1}^{(n)}}{VS_t^{(n)}}\right) = \Pi_0^{(n)} + \Pi_1^{(n)}y_t^{(1)} + \Pi_2^{(n)}f_t^{(2)} + \Pi_3^{(n)}f_t^{(3)} + \Pi_4^{(n)}f_t^{(4)} + \varepsilon_{t+1}^{(n)}, \quad (14)$$

for $n=1,2,3,4$. The variance swap rates are computed each month (see, e.g., Carr and Wu, 2009) as

$$VS_t^{(n)} = \frac{365}{\tau_n} \int_{K > S_t} \frac{2e^{r^* \tau_n}}{K^2} C_t^{(n)}[K] dK + \frac{365}{\tau_n} \int_{K < S_t} \frac{2e^{r^* \tau_n}}{K^2} P_t^{(n)}[K] dK, \quad n = 1,2,3,4, \quad (15)$$

where, as before, τ_n denotes the number of days to maturity. Understanding the behavior of variance swap rates is important, as they are often a vehicle for taking volatility bets, and/or immunizing existing positions to the risk in market volatility (Biscamp and Weithers, 2007; Carr and Lee, 2008; Andersen and Benzoni, 2009).

The empirical results reported in Table 10 tell us that a forward variance $f_t^{(n)}$ tends to be a significant predictor of the changes in variance swap rates of maturities adjacent to the n -th maturity. For example, the coefficient on $f_t^{(2)}$ is negative and statistically significant, with individual Hodrick p -values below 0.046 in the regressions involving $q_{t+1}^{var\ swap(1)}$ and $q_{t+1}^{var\ swap(2)}$, but loses its significance when predicting $q_{t+1}^{var\ swap(3)}$ and $q_{t+1}^{var\ swap(4)}$. Hence, an increase in the 30-day variance anticipated between 19 and 49 days

Table 10

Predicting log changes in variance swap rates.

At each date t , the annualized variance swap rate with maturity τ_n , for $n=1,2,3,4$, is computed as: $VS_t^{(n)} = (365/\tau_n) \int_{K > S_t} (2e^{r^* \tau_n}/K^2) C_t^{(n)}[K] dK + (365/\tau_n) \int_{K < S_t} (2e^{r^* \tau_n}/K^2) P_t^{(n)}[K] dK$, where $C_t^{(n)}[K]$ and $P_t^{(n)}[K]$ are prices of a call and put on the S&P 500 index with maturity τ_n and strike price K (data source: OptionMetrics). Reported are coefficient estimates and two-sided p -values for the following regressions over the sample period 09/1998 to 09/2008:

$$q_{t+1}^{var\ swap(n)} \equiv \ln\left(\frac{VS_{t+1}^{(n)}}{VS_t^{(n)}}\right) = \Pi_0^{(n)} + \Pi_1^{(n)}y_t^{(1)} + \Pi_2^{(n)}f_t^{(2)} + \Pi_3^{(n)}f_t^{(3)} + \Pi_4^{(n)}f_t^{(4)} + \varepsilon_{t+1}^{(n)}.$$

To correct for autocorrelation and heteroskedasticity, we use the Newey–West estimator with automatically selected lag, as in Newey and West (1994), and the reported two-sided p -values are denoted as NW-lag, p -val. Shown also are Hodrick (1992) two-sided p -values (denoted H, p -val), and two-sided p -values from the parametric bootstrap (denoted as PB, p -val). Adjusted R^2 is reported as \bar{R}^2 and the Durbin–Watson statistic is shown as $[DW]$. The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. All variables are measured in monthly units.

Dependent variable		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	\bar{R}^2 [DW]	Joint p -val
$q_{t+1}^{var\ swap(1)}$	Coef.	0.143	-23.249	-60.413	28.143	30.637	4.36%	
	NW-6, p -val	0.147	0.294	0.033	0.148	0.021	[2.06]	0.001
	H, p -val	0.204	0.357	0.045	0.189	0.059		0.113
	PB, p -val	0.146	0.422	0.146	0.379	0.089		0.112
$q_{t+1}^{var\ swap(2)}$	Coef.	0.105	-20.688	-50.391	32.688	20.578	6.27%	
	NW-3, p -val	0.129	0.276	0.015	0.023	0.100	[2.03]	0.001
	H, p -val	0.188	0.319	0.035	0.049	0.164		0.079
	PB, p -val	0.155	0.370	0.039	0.139	0.105		0.040
$q_{t+1}^{var\ swap(3)}$	Coef.	0.176	22.340	-14.251	-72.063	29.404	19.05%	
	NW-8, p -val	0.008	0.316	0.564	0.001	0.000	[2.30]	0.000
	H, p -val	0.087	0.426	0.612	0.007	0.064		0.019
	PB, p -val	0.031	0.418	0.528	0.000	0.034		0.000
$q_{t+1}^{var\ swap(4)}$	Coef.	0.150	13.249	2.107	-19.883	-18.741	2.16%	
	NW-13, p -val	0.009	0.453	0.905	0.053	0.025	[2.07]	0.006
	H, p -val	0.021	0.567	0.912	0.115	0.253		0.120
	PB, p -val	0.047	0.512	0.918	0.177	0.164		0.298

after date t , as reflected in the option market, signals a decline in the short end of the variance swap curve.

Based on the Hodrick standard errors, two joint p -values are below 0.10, and another one is under 0.02, whereas with the Newey and West standard errors, all four joint p -values are below 0.05. This evidence implies that forward variances track variations in variance swap rates. Since both the prices $H_t^{(t,n)}$, used to construct forward variances, and the variance swap quotes are derived from particular portfolios of calls and puts, the predictability of $q_{t+1}^{\text{var swap}(n)}$ reveals that certain option portfolios contain predictive information for other sets of option portfolios.

Using a system of equations analogous to (13), we also examine joint regressor significance across the variance swap maturities. Based on the $\chi^2(12)$ statistic of 29.6 and a corresponding p -value of 0.003, as shown in Table 9 (row 11, Panel A), the null hypothesis of no predictability is rejected. As also seen from rows 12 to 15 of Table 9, the documented p -values provide evidence that broadly strengthens the case for predictability via forward variances. Therefore, the predictive ability of forward variances does not appear to be specific to an isolated financial instrument, but may be a more pervasive phenomenon.

Before closing, several stylized features of the estimation results deserve further comment. First, whereas two out of four adjusted R^2 are below 5% in Table 10, the estimation for $q_{t+1}^{\text{var swap}(3)}$ exhibits an adjusted R^2 of 19.05%. Second, $f_t^{(3)}$ seems to be an important predictor in the joint estimation with all the variance swap rates (see row 11 of Panels A and B in Table 9). Finally, individual p -values, reported in Table 10, indicate that $f_t^{(3)}$ is significant when predicting $q_{t+1}^{\text{var swap}(3)}$. Our evidence collectively indicates that forward variances appear to contain information that is relevant to instruments sensitive to integrated variance.

5. Conclusions

Examining the return variation of stocks and bonds, as well as its relation to economic fundamentals, has shaped much of our knowledge about the determinants of asset return dynamics and the sources of return predictability. The empirical treatments of Fama (1981, 1990), Fama and Bliss (1987), Fama and French (1989, 1993), Campbell and Shiller (1991), Ferson and Harvey (1991), Bekaert and Hodrick (1992), and Cochrane and Piazzesi (2005) are some examples of this strand of research. Contributing to the same line of thinking, this paper offers a framework to connect proxies for forward variance, as inferred from the option market, to subsequent returns of stock and bonds, and to growth in real economic activity.

Building on Carr and Lee (2008), this paper presents an index option positioning that furnishes the price of exponential claims on integrated variance. Key features of such claims are their investability, model-free characterization, and sensitivity to changes in variance, but not to movements of the underlying index. Furthermore, our approach allows us to extract forward variances over various periods, which are instrumental to our empirical exercise (much in the way that forward interest rates, obtained from various discount bonds, have enabled researchers to draw implications for bond

returns and risk premiums in bond markets). In our setting, forward variances are related to the variation in traded market volatility, an important asset class now available to investors.

Suggestive of their broader relevance, forward variances help predict the growth rate of non-farm payroll and industrial production, and other measures of real economic activity, implying that predictability has roots in expectations about the real economy, as perceived in the option market. Probing further, we find that forward variances predict stock market returns in the presence of traditional predictors. Forward variances are also found to be useful predictors of Treasury bill returns, suggesting a link between forward variances, stock market returns, and Treasury bill returns. Completing our empirical analysis, forward variances predict changes in variance swap rates. Predictive ability is measured by both individual and joint statistical significance of the slope coefficients within a market, as well as across a set of markets.

Possible concerns about the impact of nearly integrated predictors on inference are addressed through a simulation, which indicates that the Hodrick (1992) test offers the correct size in small samples, under empirically relevant assumptions about the data generating process of the predictors, even when some predictors are allowed to be nearly integrated. A parametric bootstrap procedure is presented as an additional safeguard in assessing the predictive significance when the sample size is small.

Finally, our empirical findings should be viewed with some caution. A limitation is that the construction of monthly forward variances requires index option data with a dense set of strikes over multiple maturities, and hence, our study is confined to the September 1998 to September 2008 sample period. At the same time, the documented interplay of forward variances across financial markets and the real economy deserves further theoretical scrutiny, and this is where we expect much demanding work to be done.

Appendix A. S&P 500 index option data at four maturities

Here, we briefly describe the S&P 500 index option data from OptionMetrics. In-the-money S&P 500 index calls and puts are omitted. In addition, we discard index options with zero traded volume, and hence avoid using matrix prices for options which have open interest but are not traded. Options which allow for arbitrage across strikes are discarded.

From the remaining option data, available at daily frequency, options at the end of each month are selected over the sample period from September 1998 to September 2008. Prior to September 1998, we were unable to find a comprehensive set of options that would allow construction of a reliable time series of prices for exponential claims on integrated variance at maturities of interest. Following convention, we use the bid-ask midpoint option quote.

Given the expiration cycle of index options at the CBOE, at the end of each month there are available traded options expiring in each of the following three months.

Table A1

Robustness of the results for real activity.

Here we replicate results from Table 2, using alternative term structure variables. Reported are coefficient estimates and two-sided Hodrick (1992) p -values (denoted H, p -val), and \bar{R}^2 is the adjusted R^2 . The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. The sample is 09/1998 to 09/2008 (121 observations). The term structure variables, included as predictors, are (i) the return difference between Moody's Baa bond and the ten-year Treasury bond, denoted by $rdef_t$, (ii) the credit spread, denoted by $credit_t$, and defined as Moody's Baa bond yield minus Aaa bond yield, and (iii) the return difference between the ten-year Treasury bond and the three-month Treasury bill, denoted by $rterm_t$ (data source: Federal Reserve Board). All variables are measured in monthly units.

Non-farm payroll								
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$credit_t$	\bar{R}^2	Joint p -val
Coef.	0.003	-0.412	0.065	0.201	0.162	-3.405	44.45%	
H, p -val	0.000	0.001	0.576	0.013	0.016	0.000		0.000
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$rdef_t$	\bar{R}^2	Joint p -val
Coef.	0.000	-0.408	0.018	0.225	0.218	0.001	22.43%	
H, p -val	0.765	0.000	0.875	0.005	0.001	0.941		0.003
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$rterm_t$	\bar{R}^2	Joint p -val
Coef.	0.000	-0.418	-0.001	0.229	0.232	0.006	23.15%	
H, p -val	0.651	0.000	0.992	0.004	0.001	0.308		0.000
Industrial production								
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$credit_t$	\bar{R}^2	Joint p -val
Coef.	0.007	-1.086	0.571	0.129	0.252	-7.066	6.02%	
H, p -val	0.162	0.007	0.170	0.669	0.224	0.220		0.034
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$rdef_t$	\bar{R}^2	Joint p -val
Coef.	0.003	-1.153	0.367	0.299	0.374	-1.954	6.84%	
H, p -val	0.315	0.004	0.315	0.231	0.033	0.476		0.054
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$rterm_t$	\bar{R}^2	Joint p -val
Coef.	0.000	-1.049	0.543	0.161	0.313	-0.021	2.20%	
H, p -val	0.824	0.011	0.175	0.58	0.101	0.408		0.048

Table A2

Robustness of the results for Treasury returns.

Here we replicate results from Table 4, using alternative term structure variables as predictors. Reported are coefficient estimates and two-sided Hodrick (1992) p -values, and \bar{R}^2 is the adjusted R^2 . The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. The sample is 09/1998 to 09/2008 (121 observations). The term structure variables, included as predictors, are (i) the return difference between Moody's Baa bond and the ten-year Treasury bond, denoted by $rdef_t$, (ii) the return difference between the ten-year Treasury bond and the three-month Treasury bill, denoted by $rterm_t$, (iii) credit spread, denoted as $credit_t$, and defined as Moody's Baa bond yield minus Aaa bond yield, and (iv) the slope of the Treasury yield curve, denoted as $yslope_t$, and defined as the ten-year Treasury yield minus the three-month Treasury yield (data source: Federal Reserve Board). All variables are measured in monthly units.

Treasury bills								
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$credit_t$	$rdef_t$	\bar{R}^2		Joint p -val
Coef.	0.000	0.044	-0.055	0.574	-0.004	10.86%		
H, p -val	0.072	0.012	0.001	0.043	0.059			0.002
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rdef_t$	$yslope_t$	\bar{R}^2		Joint p -val
Coef.	0.000	0.037	-0.054	-0.004	-0.008	2.85%		
H, p -val	0.083	0.038	0.002	0.051	0.694			0.004
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rdef_t$	$rterm_t$	\bar{R}^2		Joint p -val
Coef.	0.000	0.038	-0.053	-0.004	0.000	2.81%		
H, p -val	0.066	0.038	0.003	0.061	0.971			0.003
Treasury bonds								
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$credit_t$	$rdef_t$	\bar{R}^2		Joint p -val
Coef.	0.005	1.528	-2.077	1.145	0.165	-0.83%		
H, p -val	0.680	0.314	0.189	0.933	0.364			0.435
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rdef_t$	$yslope_t$	\bar{R}^2		Joint p -val
Coef.	0.000	1.667	-1.713	0.149	2.717	0.22%		
H, p -val	0.992	0.271	0.286	0.410	0.288			0.315
	Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rdef_t$	$rterm_t$	\bar{R}^2		Joint p -val
Coef.	0.006	1.510	-2.059	0.159	-0.010	-0.83%		
H, p -val	0.409	0.310	0.182	0.416	0.926			0.452

Table A3

Robustness of the results for the stock market.

Here we replicate results from Table 6, using alternative term structure variables. Reported are coefficient estimates and two-sided Hodrick (1992) p -values (denoted H, p -val), and \bar{R}^2 is the adjusted R^2 . The final column reports p -values for the null hypothesis that the slope coefficients are jointly equal to zero. The sample is 09/1998 to 09/2008 (121 observations). The term structure variables, included as predictors, are (i) the return difference between Moody's Baa bond and the ten-year Treasury bond, denoted by rdf_t , (ii) the credit spread, denoted by $credit_t$, and defined as Moody's Baa bond yield minus Aaa bond yield, and (iii) the return difference between the ten-year Treasury bond and the three-month Treasury bill, denoted by $rterm_t$ (data source: Federal Reserve Board). All variables are measured in monthly units.

S&P 500		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$credit_t$	$(e/p)_t$	\bar{R}^2	Joint p -val
Coef.		-0.070	-2.651	6.427	5.634	12.137	6.77%	
H, p -val		0.054	0.391	0.039	0.827	0.007		0.030
		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rterm_t$	$(e/p)_t$	\bar{R}^2	Joint p -val
Coef.		-0.063	-2.749	6.384	0.013	11.674	6.72%	
H, p -val		0.008	0.348	0.042	0.940	0.006		0.041
		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rdef_t$	$(e/p)_t$	\bar{R}^2	Joint p -val
Coef.		-0.064	-2.727	6.422	0.011	11.749	6.72%	
H, p -val		0.009	0.366	0.039	0.969	0.005		0.037
MSCI		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$credit_t$	$(e/p)_t$	\bar{R}^2	Joint p -val
Coef.		-0.055	-5.468	7.633	1.549	11.450	8.72%	
H, p -val		0.112	0.099	0.023	0.950	0.008		0.019
		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rterm_t$	$(e/p)_t$	\bar{R}^2	Joint p -val
Coef.		-0.051	-5.514	7.475	0.077	11.143	8.87%	
H, p -val		0.03	0.086	0.028	0.652	0.007		0.019
		Const.	$y_t^{(1)}$	$f_t^{(2)}$	$rdef_t$	$(e/p)_t$	\bar{R}^2	Joint p -val
Coef.		-0.052	-5.525	7.609	-0.02	11.255	8.72%	
H, p -val		0.029	0.091	0.024	0.943	0.006		0.023

At the same time, longer-maturity options are introduced at the CBOE at three-month intervals, so, where needed, we interpolate implied volatilities to obtain the prices of options with expiration in the fourth month after the end of each month in our sample. We are careful to verify that option prices obtained in this way do not allow for arbitrage and are above the minimum tick size.

We denote by τ_n the four option maturities, which are about 19, 49, 79, and 109 days, respectively, and where $n=1,2,3,4$. It is crucial to employ time series of prices $H_t^{(t,n)}$ of exponential claims on integrated variance with approximately constant time to expiration, close to the respective τ_n .

Appendix B. Statistical properties of forward variances

We examine four aspects of the data, namely (i) cross correlations, (ii) pattern of autocorrelations, (iii) stationarity, and (iv) persistence of forward variances. In our context, the presence of very persistent predictors may adversely affect the finite sample performance of predictability tests.

Panel A of Table 1 presents the cross correlations. Over the September 1998 to September 2008 sample period, the correlations between $y_t^{(1)}$, $f_t^{(2)}$, $f_t^{(3)}$, and $f_t^{(4)}$ range between 0.32 and 0.81. The documented correlations are lower than those between forward interest rates (for recent evidence, see, for example, the data set made available by Cochrane and Piazzesi, 2005), implying that

the forward variances over different horizons likely convey distinct information.

Next, the maximum first-order autocorrelation coefficient among the forward variances, denoted by $ACF(1)$, is 0.72 (see Panel B of Table 1). However, the slow decay of the autocorrelations suggests that the AR(1) model may be inadequate, and a low order ARMA(p,q) model may be needed to reproduce the documented autocorrelation patterns. We comment further on this point shortly.

Proceeding to examine stationarity, we appeal to tests of unit root from two perspectives. Panel C of Table 1 first shows the results from applying the Phillips and Perron (1988) test to the forward variances, which has the advantage of being robust to general forms of heteroskedasticity of the error term. We also present the Parker, Paparoditis, and Politis (2006) unit root test, which is based on the stationary bootstrap. The two tests agree in rejecting the null hypothesis of a unit root, and are consistent with our observations based on Fig. 1 that the forward variances appear to exhibit mean-reverting behavior.

To investigate near-unit-root behavior, we turn to a model selection exercise, starting with an AR(1) model for forward variances. Then, we consider more elaborate models in the ARMA(p,q) family, with and without GARCH effects, for low values of p and q (see, French, Schwert, and Stambaugh, 1987; Hodrick, 1992; and Andersen, Bollerslev, Diebold, and Labys, 2003). Among these models, the BIC information criterion selects an ARMA(1,1)-GARCH(1,1) model for each forward variance, with estimated AR coefficients in the range 0.874 to 0.927, and MA

coefficients in the range -0.220 to -0.667 , as displayed in Panel D of Table 1.

Note further that, while the sum of the ARCH and GARCH coefficient estimates exceeds unity for two of the forward variances, we have verified that the necessary and sufficient condition for stationarity in Nelson (1990) is satisfied for both of these two cases.

For a closer examination of the persistence of forward variances under the ARMA(1,1)-GARCH(1,1) model, we test the null hypothesis that the AR coefficient is equal to 0.96, 0.97, 0.98, and 0.99 versus the one-sided alternative. Based on the p -values reported in Panel E of Table 1, one uniformly rejects, at the 5% confidence level, the hypothesis that the AR coefficient is equal to 0.99 against the alternative hypothesis that it is less than 0.99 for each forward variance.

Overall, we view these results as indicating that the forward variances do not appear to suffer from problems associated with very persistent predictors of the type described in, for example, Valkanov (2003), Torous, Valkanov, and Yan (2004), Campbell and Yogo (2006), and Boudoukh, Richardson, and Whitelaw (2008). We reiterate here that, in our setting, the performance of the Hodrick (1992) 1B test is satisfactory, even under persistent predictors, as demonstrated via simulation in Table 8.

Appendix C. Parametric bootstrap used to compute individual and joint p -values

The purpose of this appendix is to sketch the parametric bootstrap, used for computing individual and joint p -values in our predictive regressions. The procedure is in the flavor of those employed in Mark (1995), Cochrane and Piazzesi (2005), and Romano and Wolf (2006); it is closest to the one in Amihud, Hurvich, and Wang (2009) and is adapted to our setting with ARMA(1,1)-GARCH(1,1) predictors.

For parsimony of exposition, we consider a predictive regression with two predictors, with the understanding that more predictors can be treated in a similar way. As in Amihud, Hurvich, and Wang (2009), let, $y_{t+1} = \alpha + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \varepsilon_{t+1}$. In the description that follows, we present in detail the testing of the hypothesis $\beta_1 = 0$. Similar steps apply when testing for $\beta_1 = \beta_2 = 0$.

1. Estimate the ordinary least squares (OLS) coefficients $\hat{\alpha}$, $\hat{\beta}_1$, and $\hat{\beta}_2$, and the OLS t -statistic $t_{\hat{\beta}_1}$. Store the residuals ε_{t+1} .
2. Estimate, with maximum likelihood, an ARMA(1,1)-GARCH(1,1) model on $x_{1,t}$ and $x_{2,t}$, and store the respective parameters and standardized residuals.
3. Bootstrap the standardized residuals. Using the estimated parameters and these residuals, build iteratively bootstrapped predictor series $x_{1,t}^j$ and $x_{2,t}^j$, for $j=1, \dots, N$. We use $N=50,000$.
4. Build bootstrapped series of the predicted variable under the null $\beta_1 = 0$, as $y_{t+1}^j = \hat{\alpha} + \beta_2 x_{2,t}^j + \varepsilon_{t+1}^j$, where ε_{t+1}^j are bootstrap samples of the regression residuals ε_{t+1} .
5. The bootstrap in Steps 3 and 4 above is performed in a pairwise manner: we form a matrix from the original

standardized residuals and regression residuals, and then resample, with replacement, the rows of this matrix to build the bootstrapped predictors and predicted variable. This procedure preserves the dependencies between regression residuals and predictors. Furthermore, the pairwise bootstrap has been shown to be consistent with possibly heteroskedastic regression residuals (e.g., MacKinnon, 2006).

6. Next, estimate the full regression on the bootstrapped variables: $y_{t+1}^j = \alpha^j + \beta_1^j x_{1,t}^j + \beta_2^j x_{2,t}^j + u_{t+1}^j$, and store the OLS t -statistics $t_{\hat{\beta}_1^j}$.
7. Finally, compute the proportion of $|t_{\hat{\beta}_1^j}|$ that exceed $|t_{\hat{\beta}_1}|$ from Step 1. This proportion is the bootstrapped p -value of β_1 .

Testing for $\beta_1 = \beta_2 = 0$ only differs by (i) obtaining, in Step 1, a χ^2 statistic for the joint test of parameter insignificance, instead of $t_{\hat{\beta}_1}$, and (ii) generating, in Step 4, the predicted variable under the null as $y_{t+1}^j = \hat{\alpha} + \varepsilon_{t+1}^j$. The p -values are reported in the row marked "PB, p -val" in the tables.

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