The Real Determinants of Asset Sales

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ABSTRACT

I develop a dynamic structural model in which a firm makes rational decisions to buy or sell assets in the presence of productivity shocks. By identifying equilibrium asset prices, the model also examines the aggregate asset sales activity over the business cycle. It shows that changes in productivity, rather than productivity levels, affect decisions: Firms with rising productivity buy assets and firms with falling productivity downsize (“rising buys falling”). As such, industries in which firms have less persistent and more volatile productivity experience greater asset reallocation. Using plant-level data from Longitudinal Research Database (LRD), I find strong support for the model’s predictions.

ONE OF THE PUZZLES IN CORPORATE FINANCE is why asset sales and merger activities vary not only across industries, but also over time. Many researchers find similar results: Assets are traded much more frequently in some industries than others, and a greater number of transactions occur in expansion years when aggregate productivity is high. But, what makes assets in one industry more likely to change hands than assets in others? Why do asset purchases and sales coincide with business cycles? Which firms engage in those transactions?

This paper addresses these questions by providing a unified framework that jointly analyzes firms’ decisions to buy or sell assets and the activity of asset reallocation in the industry. In the model, firms experience both firm- and industry-level productivity shocks, and they maximize value by making two interrelated decisions: how much to invest and whether to buy or sell assets. Asset prices are endogenous from market clearing.

I show that asset sales are driven by changes in productivity brought by shocks. It is optimal for a firm to expand when productivity rises, and to

*Yang is at UCLA Anderson School. This paper is part of my doctoral dissertation at University of Maryland. I wish to thank my advisors, Vojislav Maksimovic (Chair), Gordon Phillips, Nagpurnanand Prabhala, Pete Kyle, and John Rust, for their encouragement and helpful discussions throughout my dissertation. Also, I would like to acknowledge helpful comments from Antonio Falato, Gerard Hoberg, Mark Loewenstein, Mark Chen, Steve Heston, Gurdip Bakshi, and Liang Zuo; seminar participants at University of Maryland, University of Arizona, Cornell University, Southern Methodist University, University of Texas at Dallas, University of Washington, UCLA, Virginia Tech, University of Minnesota, the 2006 China International Conference in Finance, and the 2006 FMA Conference; and especially an anonymous referee, and the editor Cam Harvey. All remaining errors are mine. The research in this paper was conducted while the author was a Special Sworn Status researcher of the U.S. Census Bureau at the Center of Economic Studies. Research results and conclusions expressed are those of the author and do not necessarily reflect the views of the Census Bureau. This paper has been screened to ensure that no confidential data are revealed.
downsize when productivity falls. The decision to invest internally or acquire externally depends on the relative price between existing and new assets. When the productivity increase is driven by the aggregate shock, asset prices go up as all firms become more productive, and that makes investments more attractive than buying assets. On the other hand, if the rise of productivity is driven by firm-specific shocks, it is usually optimal for firms to buy assets from other firms that wish to downsize due to falling productivity. Although asset reallocation is mainly driven by firm-specific shocks and new investments are most affected by industry shocks, two decisions are also connected. A positive aggregate shock can boost asset transfers when gains from transferring assets are bigger, and a negative aggregate shock may draw firms in need of additional capacity to the market of existing assets as asset prices become cheaper due to excess supply.

The main intuition of the model yields the following corollary results. First, changes in productivity, rather than productivity levels, affect a firm's decision to buy or sell assets. This prediction comes directly from the concavity of the production function and the dynamic aspects of the model. Every period, a firm chooses the optimal size given its productivity. Adjustments are only necessary when changes occur. Therefore, the buyer can have higher or lower productivity than the seller in a value-maximizing transaction. The focus on change introduces a new perspective to debates in the literature regarding relative productivity levels between the buyer and the seller in acquisitions (Jovanovic and Rousseau (2002), Rhodes-Kropf, Robinson, and Viswanathan (2005), and Rhodes-Kropf and Robinson (2008)).

Second, industries in which firms have less persistent productivity (more frequent changes) and more volatile shocks (bigger changes when they occur) have higher intensity in asset reallocation. Using plant-level data from the Longitudinal Research Database (LRD) maintained by the U.S. Census Bureau in the period between 1974 and 2000, I find strong support for the model's predictions. After controlling for other industry characteristics, shock properties such as persistence and dispersion explain about 16% of the industry fixed effects in asset sales.

Third, when there exist fixed transaction costs, in equilibrium more asset reallocation takes place in expansion years when the gain from transferring assets is higher and marginal buyers and sellers are able to trade. This finding provides an alternative explanation for the evidence that asset reallocation is pro-cyclical due to lower financing costs or market overvaluation (Eisfeldt and Rampini (2006), Shleifer and Vishny (2003), and Rhodes-Kropf et al. (2005)).

I solve the model in a recursive equilibrium framework through simulations for firms' investment decisions and the equilibrium asset prices implied by those decisions in an industry with heterogeneous firms. The model is then

1 Jovanovic and Rousseau (2002) argue that firms with high productivity buy firms with low productivity ("high buys low"); Rhodes-Kropf et al. (2005) suggest that lower long-run M/B firms buy higher long-run M/B firms ("long-run low buys long-run high"); while Rhodes-Kropf and Robinson (2008) show evidence that mergers typically pair together firms with similar M/B ratios ("like buys like").
calibrated by matching the simulated moments with data moments. Using the simulated panel from the calibrated model, I reexamine two strands of recently documented empirical evidence on asset sales.

First, I show that a dynamic neoclassical model with productivity shocks generates transaction patterns that were previously viewed as support for the misvaluation theory, as documented by Rhodes-Kropf et al. (2005). They show that acquirers have higher valuation around transactions but lower long-run average valuation while targets have lower valuation around transactions but higher long-run valuation. In my model, similar patterns occur as a result of equilibrium behavior without misvaluation. The intuition is as follows. As productivity fluctuates, firms with rising productivity expand and firms with falling productivity downsize. Hence, overall, buyers are more productive than sellers. In addition, since productivity follows a mean-reversion process and eventually moves toward its long-run mean, firms are more likely to acquire (sell) assets at times when their current productivity is above (below) their long-run average. Therefore, controlling for current valuations, buyers have lower long-run valuation than do sellers. The phenomenon of “long-run low buys long-run high” can be caused by productivity shocks rather than misvaluation.

Second, I show that consistent with Moeller, Schlingemann, and Stulz (2004), small acquirers have higher returns around acquisitions than do large acquirers. This is because with fixed transaction costs, acquisitions are more likely when productivity increases are high and firm sizes are large. Both of these factors lead to higher payoffs in dollar terms to outweigh the fixed transaction costs. Therefore, small acquirers demand bigger increases in productivity to take on transactions, and their higher returns are reflections of those increases.

This paper is closely related to the work of Gort (1969), Maksimovic and Phillips (2001, 2002), and Jovanovic and Rousseau (2002). Although my model does incorporate features from these models, it differs in several crucial ways. First, different from the model in Jovanovic and Rousseau (2002), where asset sales are driven by cross-sectional differences in productivity, I allow firms of different productivity to coexist and model their decisions to change capacity based on the existing productivity level as well as shocks. Second, the main driver for asset purchases and sales in this model is idiosyncratic shocks that affect the relative positions within the industry, that is, the shuffling effect. This is very different from Maksimovic and Phillips (2002), who model asset reallocation as firms’ responses to aggregate demand shocks. As such, I am able to model asset sales when the aggregate state is constant or in transition and explain the cross-industry variation in asset reallocation. To my knowledge, this is the first paper that studies asset sales in a dynamic framework with equilibrium asset prices and a distribution of heterogeneous firms. The dynamic setting allows me to address firm decisions over time, and the equilibrium framework makes it possible to link firm decisions with aggregate industry activities.

Many empirical studies document how economic shocks affect asset reallocation (Mitchell and Mulherin (1996), Andrade, Mitchell, and Stafford (2001),
Andrade and Stafford (2004) and Harford (2005)). Other studies explain how asset sales can be related to firm characteristics. For example, Jensen (1986) provides an agency explanation for asset purchases. Lang, Poulson, and Stulz (1995), and Schlingemann, Stulz, and Walking (2002) focus on liquidity and financing needs. Warusawitharana (2008) predicts asset sales using profitability and firm size. This paper contributes to the literature by providing new evidence on how dynamic properties of productivity shocks may affect asset purchases and sales. The equilibrium aspect of my model is in the spirit of Shleifer and Vishny (1992). These authors demonstrate that in recessions, existing assets are traded at a discount due to liquidity constraints. Here, I show that existing capital can be traded at a discount or premium compared to the price of new investments even without liquidity constraints. In addition, this paper joins a small but growing literature in corporate finance that matches the simulated panel based on structural models to an empirical panel to recover firms’ decisions (Gomes and Livdan (2004), Hennessy and Whited (2005, 2007), and Strebulaev (2007)). The structural approach provides a useful solution to the endogeneity problems embedded within most empirical studies, which as Coles, Lemmon, and Meschke (2005) show, are difficult to correct by using the standard econometric methods.

The rest of the paper is organized as follows. Section I describes the model. Section II discusses the calibration method and the model’s implications. Section III reconciles theory with empirical evidence. Section IV presents model predictions and results from empirical tests, and Section V concludes.

I. The Model

A. Firms

There is only one industry in the economy, firms produce a common good using only capital, and the price of this common good is normalized to one. Firms are funded fully with equity. All proceeds are paid out and there is no cost of raising capital or distributing dividends.

At the beginning of any time $t$, firm $i$ is endowed with a capital stock $(k_{i,t})$. The firm observes both an industry productivity $(z_{a,t})$ and a firm productivity $(z_{i,t})$, and decides whether to engage in new investments $(I_{i,t})$, and whether to buy or sell assets $(x_{i,t})$. A positive $x_{i,t}$ denotes a purchase and a negative $x_{i,t}$ denotes a sale. There is no short selling on new investments $(I_{i,t} \geq 0)$. Firms cannot sell more than what they have in hand $(k_{i,t} + x_{i,t} \geq 0)$.

There is a timing difference between investing in new assets and buying assets from other firms. Purchased assets can be used for production in the current period, but it takes one period for new investment to become productive. The difference in timing reflects the time-to-build attribute of capital investments, that is, new machinery and structures usually take several periods to

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$^2$An earlier version of the paper containing a two-firm example for basic intuition can be downloaded from the author’s SSRN webpage.
become productive, while acquisition of another firm’s assets can be achieved in a relatively short time.³⁴

After investment decisions are made, the firm produces output using the available capital stock, \( k_{i,t} + x_{i,t} \), via an increasing and concave production function, \( \pi() \):

\[
\pi(k_{i,t} + x_{i,t}, z_{a,t}, z_{i,t}) = \exp(z_{a,t} + z_{i,t})(k_{i,t} + x_{i,t})^\alpha,
\]

where \( 0 < \alpha < 1 \). The capital carried over to the next period is equal to the undepreciated portion of previous capital plus the new investment, \((1 - \delta)(k_{i,t} + x_{i,t}) + I_{i,t}\).

There is a fixed cost, \( f \), for participating in asset transfers. This cost accounts for transaction costs such as searching for partners, negotiating deals, and legal fees.⁵ In addition, new investment is assumed to be bounded from both below and above. Investments are not reversible, and they cannot be greater than a certain proportion of current capital stock, \( 0 \leq I_t \leq I_{\max} \times k_t \). The upper bound on investment reflects the limited supply of new investments in the short run.⁶

Figure 1 illustrates the timing of events.

Quadratic adjustment costs exist for both types of investments:

\[
\Gamma_x[k_t, x_t] = \frac{\gamma_x}{2} \left( \frac{x_t}{k_t} \right)^2 k_t
\]

\[
\Gamma_f[k_t, I_t] = \frac{\gamma_f}{2} \left( \frac{I_t}{k_t} \right)^2 k_t.
\]


⁴ Although the time-to-build assumption on new investments can be reasonably justified, the main results in the paper hold without this constraint on either type of asset. Since asset sales are the only way to “release” capacity beyond depreciation, firms in need of downsizing will always set the price so that expanding firms are indifferent between buying existing assets or investing in new assets. The choice between two types of investments, on the other hand, does depend on the time-to-build assumption.

⁵ Since asset reallocation involves coordination between buyers and sellers, it is assumed to involve transaction costs in addition to those that arise in new investments.

⁶ See Jovanovic (1998) who discusses the supply constraint for new capacity in the short run.
The coefficients on adjustment costs describe how costly it is for the firm to adjust capacity. Since accommodations must be made when additional assets are bought or when existing capital is diverted, I assume that adjustment costs apply to both purchases and sales.7

The unit price of new capital is normalized to one and I denote the price of existing capital as $P$. The cash flow at period $t$ becomes

$$D[k_t, z_i, t; z_a, t, P_t] = \exp(z_{a,t} + z_{i,t})(k_t + x_t)^\alpha$$

$$- \left( \begin{array}{c}
1_{x \neq 0} f + P_t x_t + \Gamma_x[k_t, x_t] + I_t + \Gamma_I[k_t, I_t]
\end{array} \right),$$

which equals the production profit minus the costs/proceeds of asset transfers and the costs of new investment.8

Both industry and firm-specific productivity follow AR(1) processes:

$$z_{a,t+1} = \rho_a \cdot z_{a,t} + \varepsilon_{a,t+1}$$

$$z_{i,t+1} = \rho_i \cdot z_{i,t} + \varepsilon_{i,t+1},$$

where $\varepsilon_{a,t}$ and $\varepsilon_{i,t}$ are normal random variables with mean zero and variance equal to $\sigma_a^2$ and $\sigma_i^2$, respectively. At each period, $z_{a,t}$ and $z_{i,t}$ are independent of each other and the $z_{i,t}^j$ are independent across all firms.

To simplify notation, I drop the index $t$ henceforth. Consider a firm with $k$ units of installed capital at the beginning of a period and firm-specific productivity $z_i$. The firm faces an industry productivity $z_a$, and asset price, $P$. The firm chooses the amount of new investment $I$, and an asset transfer strategy, $x$, to maximize its current cash flow plus its discounted expected future firm value. The optimization behavior of this firm can be summarized by a value function $V(k, z_i; z_a, P)$, which solves the following dynamic programming problem:

$$V[k, z_i; z_a, P] = \max_{0 \leq I \leq k, x \leq -k} \left\{ D[k, z_i; z_a, P] + \beta E\left\{ V[k', z_i'; z_a', P' | z_i; z_a, P]\right\} \right\}$$

where $0 < \beta < 1$ is the intertemporal discount factor and $k', z_i', z_a', P'$ denote the values of $k, z_i, z_a, P$ at the beginning of the next period.

7 Since the firm-specific component is irreversibly lost during a transaction, both the acquirer and the seller face additional costs. The acquirer has to build its own firm-specific components into the purchased assets and the seller can only receive compensation for the nonfirm-specific portion of the assets. For a more detailed discussion on adjustment costs, see Cooper and Haltiwanger (2006).

8 When $x > 0$, this part describes the total costs of asset purchase. When $x < 0$, it represents the proceeds from asset sales, net of transaction costs.
B. The Industry

Since a firm can be described by its beginning capital and productivity \((k, z_i)\), I can summarize the industry (in addition to the aggregate productivity, \(z_a\)) by a measure \(F[K, Z_I]\) that captures the joint distribution of capital and firm-level productivity for all firms in the industry, that is, \(F = \{(k_1^1, z_1^1), (k_2^2, z_1^2), \ldots, (k_n^n, z_1^n)\}\), where \((k_j^j, z_j^j)\) represents information of firm \(j\), and \(n\) is the total number of firms in the industry. I refer to \(F\) as the industry structure henceforth.9

From (5), I can write a firm’s decisions as

\[
\begin{align*}
x &= x[k, z_i; z_a, P] \\
I &= I[k, z_i; z_a, P]
\end{align*}
\]

and its capital stock in the next period as

\[
k' = (1 - \delta)(k + x) + I = k'[k, z_i; z_a, P].
\]

Let \(X\) denote the net aggregate demand for existing capacity. Market clearing requires that the equilibrium price, \(P\), satisfies the following condition:

\[
X[P | F, z_a] = \sum_{(k, z_i) \in F} x[k, z_i; z_a, P] = 0.
\]

That is, if all firms choose optimally, the market for existing capacity will clear given the equilibrium price \(P\). Hence, I can write the equilibrium asset price, \(P\), as a function of \(F\) and \(z_a\):

\[
P = P[F, z_a].
\]

Given (7) and (9), the future industry structure can be written as a function of the current industry structure and the aggregate productivity:

\[
F' = H[F, z_a].
\]

Further, the law of motion for \(P\), the equilibrium price, can be written as

\[
P' = P[F', z_a'] = P[H[F, z_a], z_a'].
\]

That is, the future asset price depends on the current industry structure and current and future aggregate productivity.

9 Note that the marginal distribution of \(z_i\) is exogenous, whereas the capital level, \(k_i\), is endogenous.
Using (9) and (10), the value function in (5) can be rewritten as

\[
V[k, z; z', F] = \max_{0 \leq I \leq I_{\text{max}} \times k} \pi[k + x; z, z']
\]

\[= - \left( I + P[F, z_a]x + \sum_{j \in \{I, x\}} \Gamma_j[k, j] + 1_{x \neq f} \right) \]

\[+ \beta \int \int V[k', z_i'; z_a', F'] dN(z_i' | z_i) dN(z_a' | z_a), \tag{12} \]

where \( k' = (1 - \delta)(k + x) + I, F' = H[F, z_a], \) and \( N(.) \) is the cumulative distribution function of a normal distribution. Equation (12) suggests that the value of a firm equals its current profit from production (the first line) minus the investment costs (the second line) plus its discounted future value (the third line).

I characterize a recursive competitive equilibrium using the following definition:

**Definition 1 (Recursive Competitive Equilibrium):** A recursive competitive equilibrium is a set of decision functions \( \{x[.], I[.]\} \), a price function \( P[.], \) and a law of motion \( H[.], \) such that:

(a) \( V[k, z; z', F] \) solves a firm’s optimization problem in (12) given \( H[.], \) and \( P[.], \)

(b) \( P[F, z_a] \) satisfies the market clearing condition in (8)

(c) \( H[.], \) is generated by the decision rules implied by \( V[.], \)

In the equilibrium, given current aggregate productivity and industry structure, the evolution of the industry structure and the equilibrium asset price are anticipated by all firms in the industry. Based on this expectation, firms make investment decisions to maximize their value (Definition 1a). As all firms optimally adjust their capacity, the realized asset price appears to be consistent with firms’ earlier perception (Definition 1b), and so does the realized industry structure (Definition 1c).

Assuming that the fixed cost condition is not binding, the first-order conditions from (12) can be derived as follows:

\[
FOC(x): \frac{\exp(z_a + z_i)\omega(k + x)^{\omega - 1}}{\text{marginal product}} = \left( P + \frac{\partial V[k, x]}{\partial x} \right) - \beta(1 - \delta) \frac{\partial E[V[k', z_i'; z_a', F']]}{\partial k'}
\]

\[
FOC(I): \beta \frac{\partial E[V[k', z_i'; z_a', F']]}{\partial k'} = 1 + \frac{\partial V[I, k]}{\partial I} - \mu_1 + \mu_2,
\tag{13} \]

where \( \mu_1 \) and \( \mu_2 \) are the Lagrange multiplier for the nonnegativity constraint (\( I \geq 0 \)) and the upper bound constraint of new capital (\( I \leq I_{\text{max}} \times k \)), respectively.
Firms trade assets so that the current marginal product is equal to the current marginal cost (price plus adjustment costs), net of the expected resale value of the undepreciated capital. Meanwhile, firms make new investments to equate the expected future marginal value of capital to the current marginal cost of new investments.

When the fixed cost constraint does bind, that is, the net gain from buying (selling) assets from (to) other firms for more efficient usage does not outweigh transaction costs, firms do not participate in transactions despite their productivity differences. On aggregate, the higher the transaction costs, the more heterogeneity the industry presents in terms of marginal productivity of capital.

It is worth noting that decisions on asset transactions and new investments are interrelated. With decreasing return-to-scale technology, current purchases or sales affect the marginal value of assets in the future, and therefore influence decisions on new investments. Further, firms make asset purchase and sale decisions based on current productivity while they invest in new capital based on expected future productivity. In other words, given the timing difference between existing assets and new investments, firms use asset markets as a short-run fix and invest in new capacity for long-run planning.

C. Numerical Solution

To solve for the value function in (12), I face two challenges. First, one of the state variables, industry structure \(F\), is a high-dimension object. Numerical solutions to dynamic programming problems become increasingly difficult as the size of the state space increases. Second, for the recursive equilibrium, I need to derive a pricing function, \(P[F, z_a]\), and a law of motion for industry structure, \(F' = H[F, z_a]\), which, when taken by all firms in the industry, can be implied by the optimal decisions of heterogeneous firms in that industry.

I follow the methods of Krusell and Smith (1997, 1998) to derive solutions. First, to deal with the high-dimension problem, I assume that firms perceive the law of motion for mean capacity as depending only on limited moments of \(F\), and that current asset prices depend only on the same set of moments. It turns out that, similar to Krusell and Smith (1997), in my model, the first moment of \(F\) is sufficient to obtain very small prediction errors. Therefore, instead of using the entire distribution, that is, the sequence of \(\{k^j, z^j_i\}_{j=1}^N\), I can use the first moment of \(F\), \((k, z_i)\), to solve for the approximate equilibrium. Since \(z_i's\) are mean zero, the approximate law of motion for industry structure and the pricing function can be written using the mean

\[ F(K, Z_I) = \{(k^1, z_i^1), (k^2, z_i^2), ..., (k^N, z_i^N)\} \text{ has } 2N \text{ dimensions, where } N \text{ is the total number of firms in the industry.} \]

\[ \text{See Amman, Kendrick, and Rust (1996).} \]
capacity, $\bar{k}$:

$$F' = H(F, z_a) \approx \bar{H}(\bar{k}, z_a) \quad (14)$$

$$P(F, z_a) \approx \bar{P}(\bar{k}, z_a). \quad (15)$$

Second, I use an iterative approach to solve for an approximate recursive equilibrium. I start out by assuming that aggregate productivity, $z_a$, has only two levels, expansion ($z_a^H$) and recession ($z_a^L$), and that both $\bar{H}$ and $\bar{P}$ are log-linear described by a set of parameters $(a_0, a_1, b_0, b_1)$ and $(c_0, c_1, d_0, d_1)$, respectively.\textsuperscript{12}

\begin{equation}
\log \bar{k}' = \begin{cases} 
a_0 + a_1 \log \bar{k} & \text{if } z_a = z_a^L \\
b_0 + b_1 \log \bar{k} & \text{if } z_a = z_a^H \end{cases} \quad (16a)
\end{equation}

\begin{equation}
\bar{P}(\bar{k}, z_a) = \begin{cases} 
c_0 + c_1 \log \bar{k} & \text{if } z_a = z_a^L \\
d_0 + d_1 \log \bar{k} & \text{if } z_a = z_a^H \end{cases}. \quad (16b)
\end{equation}

Next, I solve for firms’ decision rules and simulate a panel of firms over a long period of time based on those decision rules. In every period, I find the asset price that clears the market and the mean capacity in the next period. Then, using the collected time series of mean capacities and asset prices, I reestimate the coefficients in (16a) and (16b), compare them with their initial values, and iterate until convergence.\textsuperscript{13}

Two things are worth mentioning. First, convergence in the parameters above would indicate that the simulated time series closely match those perceived by firms and that the obtained outcome reflects a very small deviation from perfectly rational expectations. Hence, with convergence we would have a candidate approximate equilibrium. Second, a good fit in (16a) and (16b) implies that firms that perceive these simple law of motion and pricing functions make very small mistakes compared to using the correct law of motion or pricing function, which are nonlinear functions based on the entire distribution.

Using 3,000 firms and 500 time periods in the simulation, I obtain convergence and achieve very good fit in both $\bar{H}$ and $\bar{P}$: the $R^2$ of estimating (16a) is 99%, and the $R^2$ of estimating (16b) is 95%. This approximate aggregation occurs because when there exist a large number of competitive firms, the mean of the distribution is sufficient to summarize the aggregate behavior. Appendix B offers a detailed description of the computational strategy.

\textsuperscript{12} The functional form is chosen based on Krusell and Smith (1997, 1998). They show that a log-linear specification based on the first moment provides extremely good fit in forecasting future mean capacity and market-clearing prices and that the improvements from the inclusion of higher moments or using more flexible nonlinear function forms are minuscule in quantitative terms.

\textsuperscript{13} I measure the convergence based on the norm of the difference between the perceived parameters $(a_0, a_1, b_0, b_1, c_0, c_1, d_0, d_1)$ and the estimated parameters $(\hat{a}_0, \hat{a}_1, \hat{b}_0, \hat{b}_1, \hat{c}_0, \hat{c}_1, \hat{d}_0, \hat{d}_1)$.
Substituting mean capacity $\bar{k}$ as a state variable for industry structure $F$, I can rewrite the value function of the firm as

$$V[k, z_i; z_a, \bar{k}] = \max_{0 \leq I \leq I_{\text{max}} \times k, x \geq -k} \pi[k + x, z_a, z_i]$$

$$- \left( I + \hat{P}[k, z_a]x + \sum_{j \in \{I, x\}} \Gamma_j[k, j] + 1_{x \neq 0}f \right)$$

$$+ \beta \int \int V[k', z_i'; z_a', \bar{k}] dN(z_i' | z_i) dN(z_a' | z_a),$$

where $k' = (1 - \delta)(k + x) + I, \bar{k}' = \hat{H}[k, z_a]; \hat{H}$ and $\hat{P}$ are identified by (16a) and (16b), respectively; and $N(.)$ is the cumulative distribution function of a normal distribution.

**Proposition 1:** There exists a unique function $V[k, z_i; z_a, \bar{k}] : K \times Z_I \times Z_A \times \bar{K} \to R_+$ that solves the dynamic program in (17) and generates unique optimal policy functions $I[k, z_i; z_a, \bar{k}]$ and $x[k, z_i; z_a, \bar{k}]$.

**Proof:** See Appendix A.

### II. Calibration and Simulation

#### A. Calibration

I use plant-level data from the LRD for my calibration. The LRD is a large micro database maintained by the Center for Economic Studies at the Bureau of the Census. It contains information for approximately 50,000 manufacturing plants in the SIC codes 2000 to 3999 in both the private and public sectors.¹⁴ Through separate identifiers for plants and firms, I am able to track ownership changes and identify asset purchases and sales for sample plants. My sample covers the period from 1972 to 2000. Observations are deleted if total assets or total shipments are missing, zero, or negative. I aggregate plants into firm-level business segments at the three-digit SIC level and exclude segments that have less than $1$ million in the real value of shipments in 1982 dollars. This sample is the same as that used in Maksimovic and Phillips (2008).

Since most of my data are available at an annual frequency, I assume that a time period in the model corresponds to 1 year. I set several parameters according to prior literature. Based on Cooper and Haltiwanger (2006),¹⁵ I set the production technology parameter $\alpha$ to $0.592$, and the persistence and dispersion for aggregate productivity to $0.76$ and $0.05$, respectively. These parameters are also in line with Eisfeldt and Rampini (2006) and Hennessy

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¹⁴ See Maksimovic and Phillips (2001) for a detailed description of the LRD.

¹⁵ Cooper and Haltiwanger estimate their production function with productivity shocks through calibration using data in the LRD.
I transform (4) into a discrete-state Markov chain using the method in Tauchen (1986), letting $z_i$ have 10 points of support in $[-5\sigma_i/\sqrt{1 - \rho_i^2}, 5\sigma_i/\sqrt{1 - \rho_i^2}]$. I allow $z_o$ to have two levels to represent recession and expansion years. The capital stock, $k$, lies in the set of 25 points bounded by $[k_{\text{min}}, k_{\text{max}}]$, where $k_{\text{min}}$ and $k_{\text{max}}$ are the minimum and maximum capital stock when productivity is permanent. I use a set of eight points bounded by the same end points for the state space for mean capital, $\bar{k}$. Following Gomes (2001), the intertemporal discount factor, $\beta$, is set to be $1/1.065$.

For the simulation, I start with an industry of 3,000 firms in which firms are given some initial productivity randomly chosen from a normal distribution and the same amount of initial capital. I let the industry evolve for 500 periods. The first 50 periods are dropped to allow firms to work their way out to form an endogenous industry structure. Next, I estimate the remaining parameters, $\delta, \rho_i, \sigma_i, f, \gamma I, \gamma x$, by matching six model-generated moments with the corresponding data moments. Since the main results of my model pertain to asset sales and new investments over the business cycle, I select the summary statistics to best describe these investment patterns. They include the average rate of asset sales (assets in transaction/total assets) in both expansion and recession years; the average rate of new investments (new investments/total assets) in both expansion and recession years; the average amount purchased in a transaction (assets purchased/buyer’s existing assets); and the average amount sold in a transaction (assets sold/seller’s pre-sale assets). I define an industry-year to correspond to an expansion (recession) if the growth rate of industry sales has been positive (negative) for 2 consecutive years.

To compute the aggregate asset sales ratios, I do not separate within-industry sales from cross-industry sales. Substantial differences may exist between these two types of transactions and this model is more applicable to asset sales among firms in the same industry. However, excluding cross-industry sales can potentially omit transactions that are motivated by sellers’ demand to downsize due to falling productivity. Moreover, since most of the existing literature on asset reallocation does include both types of transactions when computing the aggregate ratio (Maksimovic and Phillips (2001, 2002), Eisfeldt and Rampini (2006)), including both types makes my model more comparable to other studies. The statistics on the average transaction amount in purchases and sales are taken from table III of Maksimovic and Phillips (2001) based on plants brought (sold) by buyers (sellers) in their primary industries and hence are based on within-industry asset sales only.

Table I summarizes the estimated parameter values and compares the key summary statistics generated by the model with those found in the data.

---

16 Eisfeldt and Rampini set $\alpha = 0.333, \rho_a = 0.75, \sigma_a = 0.015$; and Hennessy and Whited choose $\alpha = 0.551, \rho = 0.74$, and $\sigma = 0.123$.

17 It can be shown that when productivities are permanent, a firm chooses its capital stock such that $k = \left(\frac{1}{1-\alpha\beta(1-\alpha)}\right)^{\frac{1}{2}}$.

18 The choice of the initial capital distribution does not affect the model outcome.
Table I
Parameter Values and Summary Statistics
This table reports the parameters of the calibrated model. I pre-specify a set of parameters ($\alpha, \rho_a, \sigma_a, \beta$), and choose the remaining parameters, $\rho_i, \sigma_i, f, \gamma I, \gamma x$, to match the simulated moments with data moments found in LRD. In Panel C, Adjusted Size is calculated as the ratio of the average size of firms in the subsample over the average size in the industry, and Adjusted Tobin's $q$ is calculated as the average $q$ in the subsample divided by the average $q$ in the industry. Tobin's $q$ is measured as the ratio of market value (implied from the value function) over book value (the level of its capacity).

Panel A: Parameter Values

<table>
<thead>
<tr>
<th>Parameter (Pre-chosen)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production technology $\alpha$</td>
<td>0.592</td>
</tr>
<tr>
<td>Discount rate $\beta$</td>
<td>0.939</td>
</tr>
<tr>
<td>Industry persistence $\rho_a$</td>
<td>0.76</td>
</tr>
<tr>
<td>Industry dispersion $\sigma_a$</td>
<td>0.05</td>
</tr>
<tr>
<td>Parameter (Calibrated)</td>
<td></td>
</tr>
<tr>
<td>Depreciation $\delta$</td>
<td>0.15</td>
</tr>
<tr>
<td>Firm persistence $\rho_i$</td>
<td>0.70</td>
</tr>
<tr>
<td>Firm dispersion $\sigma_i$</td>
<td>0.30</td>
</tr>
<tr>
<td>Adjustment cost (new investments) $\gamma I$</td>
<td>0.1</td>
</tr>
<tr>
<td>Adjustment cost (asset transfers) $\gamma x$</td>
<td>0.2</td>
</tr>
<tr>
<td>Fixed cost (asset transfers) $f$</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average asset sales ratio (Expansion)</td>
<td>4.31%</td>
<td>4.29%</td>
</tr>
<tr>
<td>Average asset sales ratio (Recession)</td>
<td>3.79%</td>
<td>3.86%</td>
</tr>
<tr>
<td>Average new investment ratio (Expansion)</td>
<td>16.95%</td>
<td>17.88%</td>
</tr>
<tr>
<td>Average new investment ratio (Recession)</td>
<td>13.51%</td>
<td>13.93%</td>
</tr>
<tr>
<td>Percentage of assets sold</td>
<td>30.45%</td>
<td>28.2%</td>
</tr>
<tr>
<td>Percentage of assets bought</td>
<td>45.48%</td>
<td>53.8%</td>
</tr>
</tbody>
</table>

Panel C: Asset Buyers and Asset Sellers (Model)

<table>
<thead>
<tr>
<th></th>
<th>Buyer</th>
<th>Seller</th>
<th>Non-Action Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted size</td>
<td>1.12</td>
<td>1.45</td>
<td>1</td>
</tr>
<tr>
<td>Adjusted Tobin's $q$</td>
<td>1.08</td>
<td>0.67</td>
<td>1</td>
</tr>
</tbody>
</table>

$^a$The numbers are taken from table III of Maksimovic and Phillips (2001).

Although my model calibration does not reproduce these statistics exactly, the simulated panel is reasonably similar to its empirical counterparts.

My estimate of $\delta$ appears reasonable at 15% for manufacturing industries, and my estimates of $\rho_i$ and $\sigma_i$ are 0.70 and 0.30, respectively, which are comparable to Cooper and Haltiwanger (2006), who estimate a persistence level of 0.885 and a dispersion level of 0.30. The model suggests that idiosyncratic
productivity has a similar persistence level to aggregate productivity, but is much more volatile. The estimated adjustment cost coefficient on asset sales ($\gamma x$) is 0.2, compared to the coefficient of 0.10 for new investments ($\gamma I$). Three reasons might explain why adjustment for existing assets is more costly. First, acquired assets are available for production right away, and efforts to accommodate those assets may come at an opportunity cost of lowering the productivity of existing plants (Schoar (2002)). Second, unlike new investments, existing assets are not made-to-order and therefore may require additional disassembling costs to tailor the assets to the buyer’s specific needs (Jovanovic and Rousseau (2002)). Finally, firm capital is partially firm-specific (e.g., human capital) and the firm-specific component is irreversibly lost in asset transfers. The acquirer has to build her own firm-specific components into the purchased assets, which makes buying external assets more costly than investing in new assets. I estimate a fixed cost ($f$) of 0.45, which translates to a 5% to 6% relative cost based on the transaction amount for an average deal.19

The model also produces stylized facts similar to empirical findings on buyer and seller characteristics such as size, productivity, and Tobin’s $q$. Acquirers are slightly larger with higher productivity and Tobin’s $q$, and sellers are much bigger with lower productivity and Tobin’s $q$. For example, an average buyer is 12% larger than the average firm and has an industry-adjusted Tobin’s $q$ of 1.08, whereas an average seller is 45% larger than the average firm, and has an industry-adjusted Tobin’s $q$ of 0.67.20

B. Decisions on Asset Sales

Figure 2 describes the derived decisions on asset transfers based on current capacity, firm productivity, and industry productivity. Decisions are shown for different size quartiles. For each quartile, I plot firm-level productivity ($z_i$) on the X-axis and plot the rate of asset transfer ($x/k$) on the Y-axis. The two vertical lines mark the 25th and 75th percentile of the productivity distribution based on the firm’s beginning size.21 The solid and dotted lines represent decisions in recession and expansion years, respectively.

Firms form their decisions based on the gap between expected and realized productivity. For firms of all sizes, sales are optimal when realized productivity is much lower than expected (below the 25th percentile), and purchases are optimal when productivity appears to be much higher than expected (above the 75th percentile). Previous research has shown that buyers and sellers tend to differ in size, profitability, and investment opportunities. Here, my model suggests a new determinant—deviation from the expectation. The impact of the

19 This is calculated using the average transaction amount and the average asset prices. The average buyer buys 8.34 units and the average seller sells 7.21 units. The average asset price is about 1.08.

20 I define Tobin’s $q$ as the ratio of firm value over book value of capital, $V(k, z_i, z_a, P)/k$, and the industry-adjusted $q$ is measured as the ratio of the firm’s $q$ over the average $q$ in the industry.

21 For each size quartile, the 25th and 75th percentile of future productivity are calculated based on 1,000 simulations.
Figure 2. Optimal decision rules for asset sales. This figure illustrates the derived decision rules for asset sales ($x$), given a firm’s current capacity ($k$) and the firm-level productivity ($z_i$). Decisions are shown in four size quartiles ($Q_1$–$Q_4$). The two vertical lines mark the 25th and 75th percentile of the productivity distribution, based on beginning capacity. The solid and dotted lines represent decisions in recession and expansion years, respectively.

deviation from expectation is also studied in Strebulaev (2007) in the context of capital structure decisions. In all cases, asset transfers are more likely in expansion years.

Figure 3 describes the dynamics of a simulated firm along several dimensions such as firm-level productivity ($z_i$), firm size ($k$), rate of asset transfer ($x/k$), and investment ratio ($I/k$).

An increase in productivity is usually followed by an increase in capacity, and a decrease in productivity leads to downsizing. Asset purchases and sales are infrequent and lumpy. The firm buys assets when there is a major increase in productivity, in which case it also invests in new assets aggressively. Meanwhile,
Figure 3. **Time path of a simulated firm.** This figure illustrates a random time path of a simulated firm based on the calibrated model. For each plot, the X-axis represents time in years.
it sells off existing assets when there is a big drop in productivity, and when that happens it rarely invests in any new assets at the same time.

In the model, there is only one industry and therefore I can only consider within-industry asset transfers. If assets can be sold to firms outside of the industry at scrap value (as in Shleifer and Vishny (1992)), the excess capacity may exit the industry beyond the depreciation. Allowing asset purchases from outside firms will increase the overall transaction volume in recessions, especially during the transition period when the economy moves from an expansion to a recession, and this effect is less significant when assets in the industry are highly specialized and the scrap value is low.

C. Asset Sales in the Business Cycle

To investigate how industry asset sales activity varies with the business cycle, I simulate a stylized time path in which four periods of recession are followed by four periods of expansion and then another four periods of recession. I choose 4 years to match the frequency of the U.S. business cycles and the average expected time until a switch in aggregate productivity occurs. I use parameters from the calibrated model and 3,000 firms in the simulation. Figure 4, Panel A presents the simulated aggregate productivity, mean capacity, asset price, and rate of asset sales.

At the moment when the positive shock strikes (t = 5), the industry’s capacity is very low after four periods of recession. To respond to the increase in aggregate productivity, investment shoots up, reaching 20% in the first year and 17% in the second year. Mean capacity rises sharply, although at a decreasing rate. The price of the existing assets also jumps up from $1.08 to $1.12 at the transition, reflecting the higher opportunity cost of capital. More assets are traded. As expansion continues and firms expand over time, new investments start to slow down and asset prices start to fall. Mean capacity, asset prices, and asset sales activity slowly move towards their long-run equilibria. When the negative productivity shock hits (t = 9), the industry is at its highest capacity, after a 4 year expansion. At the onset of the shock, new investments shrink and general demand to downsize drives asset prices down from $1.10 to $1.06. As firms keep reducing their capacity, asset prices slowly rise.

The simulation above illustrates how investment patterns change with aggregate productivity. Both new investments and asset sales are higher in expansion years, and they both peak at the moment when the economy moves from recession to expansion. The model also predicts that the price for existing assets hits its highest level when the economy comes out of a recession, at which time demand for assets is strong, and that it drops to the lowest level when the

---

22 Since outside firms cannot achieve the first-best use of those assets, they can never compete with firms in the industry in expansions when asset prices are high. Therefore, the scrap value only binds in recessions.

23 Similar assumptions are used in Eisfeldt and Rampini (2006).

24 The unit price for existing assets is above one, the unit price for new assets due to the time difference in availability between existing assets and new investments.
Figure 4. Asset sales in the business cycle. This figure illustrates the simulated aggregate asset sales activity in the business cycle. The simulation is based on the calibrated model using 3,000 firms. In Panel A, aggregate productivity is set such that four consecutive negative shocks are followed by four positive shocks and then by four negative shocks. In Panel B, a random path of 30 periods is generated.
the economy moves to a recession after a long period of expansion in response to a large amount of excess capacity.

Panel B presents similar graphs, but using a random path of 30 periods. When aggregate states switch frequently, asset prices and sales volume may not be able to fully adjust, but the intuition is qualitatively the same.

III. Reconciling Theory and Evidence

A. Valuation and Asset Sales Decisions

Recent studies suggest that mergers might be caused by misvaluation, in which case managers use overvalued equity as cheap currency to purchase assets (Shleifer and Vishny (2003), Rhodes-Kropf and Viswanathan (2004)). To support the misvaluation theory, the empirical study by Rhodes-Kropf et al. (2005) shows that the probability of being an acquirer is higher when the firm has a high current market-to-book value, but low long-run average.

In the context of this model, I perform a similar analysis using the simulated panel of firms. I define market-to-book, the long-run market-to-book, and mean market-to-book as follows:

\[
MTB_{i,t} = V(k_{i,t}, z_{i,t}, z_{a,t}, \bar{F}_t)/k_{i,t}
\]

\[
\overline{MTB}_i = \frac{1}{T} \sum_{t=1}^{T} MTB_{i,t}
\]

\[
\overline{MTB}_t = \frac{1}{n} \sum_{i=1}^{n} MTB_{i,t},
\]

where \(T\) is the number of periods that firm \(i\) has been active and \(n\) is the number of firms in the industry.

Next, I use a specification similar to that in Rhodes-Kropf et al. (2005) such that

\[
\Pr(C_{it} = 1) = \alpha + \beta_1MTB_{i,t} + \beta_2\overline{MTB}_i + \beta_3\overline{MTB}_t + \epsilon_{it}, \quad (18)
\]

where \(C_{it}\) is firm \(i\)'s choice at time \(t\) of being an acquirer, being a seller, or taking no action.\(^{25}\)

Table II reports the results. Firms are more likely to acquire when current \(MTB\) is high but the long-run average is low; and they are more likely to sell when current \(MTB\) is low but the long-run average is high. Despite the fact that firms invest optimally and there is no misvaluation in my model, I am able to rationalize that “long-run low buys long-run high.”

The intuition is as follows. In my model, the decision to buy or sell assets depends on productivity changes. Productivity is mean-reverting—it fluctuates with random shocks, and eventually converges to the long-run average.\(^{26}\) Therefore, purchases are more likely when current productivity rises above

\(^{25}\) This specification is similar to that used in table 9, Panel B in Rhodes-Kropf et al. (2005).

\(^{26}\) The model assumes an AR(1) process for both aggregate and idiosyncratic productivity.
Table II
Valuation and Asset Sales Decisions

The table reports the estimation results of Probit models using the simulated panel of 3,000 firms and 200 periods based on the calibrated model. In column 1, the dependent variable is a dummy variable that equals one if a firm purchases assets ($x > 0$), and zero if it does not buy or sell assets ($x = 0$). In column 2, the dependent variable is a dummy variable that equals one if a firm sells assets ($x < 0$), and zero if it buys or sells assets ($x = 0$). In column 3, the dependent variable is a dummy variable that equals one if a firm is an acquirer ($x > 0$), and zero if a firm is a seller ($x < 0$). $MTB_{i,t}$ is the market-to-book ratio for firm $i$ at time $t$. It is calculated as the ratio of firm value divided by its capacity. $MTB_{i}$ is the average market-to-book ratio for firm $i$, and $MTB_{t}$ is the average market-to-book ratio for all firms at time $t$. Standard errors are reported in parentheses. *** represents significance at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>1 if Acquirer,</th>
<th>1 if Seller,</th>
<th>1 if Acquirer,</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MTB_{i,t}$</td>
<td>0.642***</td>
<td>-2.005***</td>
<td>4.020***</td>
</tr>
<tr>
<td></td>
<td>(.003)</td>
<td>(.007)</td>
<td>(.032)</td>
</tr>
<tr>
<td>$MTB_{i}$</td>
<td>-1.043***</td>
<td>1.178***</td>
<td>-2.019***</td>
</tr>
<tr>
<td></td>
<td>(.015)</td>
<td>(.019)</td>
<td>(.051)</td>
</tr>
<tr>
<td>$MTB_{t}$</td>
<td>-0.160***</td>
<td>2.569***</td>
<td>-3.054***</td>
</tr>
<tr>
<td></td>
<td>(.020)</td>
<td>(.026)</td>
<td>(.070)</td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.145</td>
<td>0.403</td>
<td>0.816</td>
</tr>
</tbody>
</table>

the long-run mean and asset sales are more likely when current productivity drops below the long-run mean. As a result, controlling for current productivity, buyers may have lower long-run productivity than sellers.

B. Size Effect in Acquirer Returns

Moeller et al. (2004) find a striking size effect in acquirer returns. Their study shows that the announcement return is about 2% higher for smaller acquirers regardless of the form of the financing and the public versus private status.

Because my model is calibrated using an annual frequency, it is difficult to capture the abnormal return in a 3-day window around the acquisition, as they have done in their paper. However, I am able to calculate the annual return for acquirers in the transaction year. I define a firm’s return in period $t$ as

$$r_t = \frac{V_t - V_{t-1}}{V_{t-1}},$$

where $V_t$ and $V_{t-1}$ represent firm value in and 1 year before the transaction year, respectively.

Table III shows acquirer returns for different size quartiles before, around, and after the transaction. Just as in Moeller et al. (2004), small acquirers have higher returns in the transaction year. The average return in the smallest size quartile is about 28.6%, compared to 18.8% for those in the largest size quartile.
The Real Determinants of Asset Sales

Table III

Size Effect in Acquirer Returns

The table reports the estimation results on the size effect in acquirer returns using the simulated panel of 3,000 firms and 200 periods. Panel A shows acquirer returns in the transaction year as well as returns 2 years, and 1 year before and after the transaction. Panel B reports current and lagged firm-level productivity for acquirers of different size quartiles. Panel C presents the regression results. In Panel C, the dependent variables in (1) and (2) are acquirer returns. Column 3 reports results from a Probit model estimation, where the dependent variable equals one if a firm acquires and zero otherwise. In column 4, the dependent variable is the percentage of assets acquired, measured as the ratio of the amount of assets acquired over the buyer's existing assets before the acquisition. I define return as the ratio of change in value from the previous period divided by the lagged firm value. In all tables, \( t \) is the year in which the acquisition occurs. Standard errors are reported in parentheses. *** represents significance at the 1% level.

<table>
<thead>
<tr>
<th>Panel A: Acquirer Return by Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size Quartiles</td>
</tr>
<tr>
<td>1 (small)</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4 (large)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Change in Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size Quartiles</td>
</tr>
<tr>
<td>1 (small)</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4 (large)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Acquirer Return and Probability of Acquisition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIZE</td>
</tr>
<tr>
<td>(0.008)</td>
</tr>
<tr>
<td>Pct. Acq.</td>
</tr>
<tr>
<td>(0.001)</td>
</tr>
<tr>
<td>Industry productivity</td>
</tr>
<tr>
<td>(0.003)</td>
</tr>
<tr>
<td>Change in productivity</td>
</tr>
<tr>
<td>(0.001)</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

What causes the size effect in acquirer returns in my model, when there is no asymmetric information, no misvaluation, and firms behave rationally? Panels B and C present the answers.

With fixed transaction costs, acquisitions are more likely when productivity increases are high and firm size is large, both of which lead to higher dollar payoffs. This generates two effects. First, for the same increase in productivity,
small firms are less likely to buy assets. A one-standard deviation decrease in size (as measured by log of total assets) leads to a 25% reduction in acquisition probability, and a firm in the first size quartile is 40% less likely to engage in an acquisition as compared to a firm that is in the last size quartile.\footnote{These estimates are based on the estimation in column 3 in Table IV, Panel C.} Second, among all acquirers, small acquirers have bigger increases in productivity, and they buy more assets on a relative basis. For example, productivity increases for acquirers in the smallest size quartile are 2.5 times as high as those for acquirers in the largest size quartile, and acquirers in the smallest size quartile buy about 62% of their existing assets, as compared to 37% purchased by acquirers in the largest size quartile. Therefore, the size effect in acquirer returns may be attributed to sample selection. That is, small acquirers demand bigger increases in productivity to make acquisitions, and their higher returns around the acquisition are reflections of those productivity increases.

In my sample, after controlling for productivity changes, the size effect disappears. Large acquirers have a slightly higher return, but the impact is minimal. The size effect due to infrequent adjustment in the presence of fixed costs has also been studied by Kurshev and Streubalaev (2006).

IV. New Predictions on Asset Sales

A. Model Predictions

The dynamic model in this paper describes an industry with time-invariant productivity shocks. In this section, I use comparative statics to examine how shock attributes affect the market of asset sales across industries.

For each related parameter, I simulate the model twice: once with a parameter value 25% above and once with a parameter value 25% below the baseline value. Table IV reports sensitivities of the simulated moments on the model’s parameters. For each parameter, I calculate elasticity as the change of moments divided by the change in underlying parameters, multiplied by the ratio of the baseline parameter over the baseline moment. Similar methods are used in Hennessy and Whited (2005).

First, I find that persistence ($\rho_i$) and dispersion ($\sigma_i$) of firm-specific productivity significantly affect asset purchases and sales. Lower persistence leads to more frequent changes and higher dispersion enlarges the impact when changes do occur. In expansion years, a 25% decrease in persistence results in a 23% increase in asset sales volume (from 4.31% to 5.28%) and a 25% increase in dispersion leads to an 88% increase in sales volume (from 4.31% to 8.09%). Similar patterns appear in recession years and for frequency of transactions.

Second, fixed costs ($f$) have a negative effect on asset purchases and sales. Higher fixed costs increase the minimum gain requirement for both buyers and sellers and result in fewer sales with a larger amount. Besides being a proxy
Table IV
Sensitivity of Model Moments to Parameter

This table presents model moments using different model parameters. The baseline parameters are: $f = 0.5$, $\rho_a = 0.76$, $\sigma_a = 0.05$, $\rho_i = 0.70$, $\sigma_i = 0.3$. For each parameter, I simulate the model twice: once with parameter value 25% above the baseline value ($+\Delta$), and once with the parameter value 25% below the baseline value ($-\Delta$). Panel A reports the simulated moments and Panel B reports the elasticities. For each parameter, I calculate the elasticity as the change of moments divided by the change in underlying parameters multiplied by the ratio of baseline structural parameter over the baseline moment.

Panel A: Simulated Moments

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>Fixed Cost ($f$)</th>
<th>Persistence ($\rho_i$)</th>
<th>Dispersion ($\sigma_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$+\Delta$</td>
<td>$-\Delta$</td>
<td>$+\Delta$</td>
</tr>
<tr>
<td>$\bar{X}(z_a = H)$</td>
<td>4.31%</td>
<td>3.73%</td>
<td>5.49%</td>
<td>2.96%</td>
</tr>
<tr>
<td>$\bar{X}(z_a = L)$</td>
<td>3.79%</td>
<td>3.13%</td>
<td>4.60%</td>
<td>2.53%</td>
</tr>
<tr>
<td>$\bar{X}(z_a = H)$</td>
<td>17.0%</td>
<td>16.4%</td>
<td>17.1%</td>
<td>16.8%</td>
</tr>
<tr>
<td>$\bar{X}(z_a = L)$</td>
<td>13.5%</td>
<td>13.4%</td>
<td>13.7%</td>
<td>13.4%</td>
</tr>
<tr>
<td>Freq ($x &gt; 0, H$)</td>
<td>7.74%</td>
<td>6.42%</td>
<td>10.32%</td>
<td>5.84%</td>
</tr>
<tr>
<td>Freq ($x &gt; 0, L$)</td>
<td>6.79%</td>
<td>6.18%</td>
<td>8.83%</td>
<td>4.58%</td>
</tr>
<tr>
<td>$E(x \mid x &gt; 0)$</td>
<td>45.5%</td>
<td>47.4%</td>
<td>42.2%</td>
<td>41.3%</td>
</tr>
<tr>
<td>$E(x \mid x &lt; 0)$</td>
<td>-30.5%</td>
<td>-31.6%</td>
<td>-29.6%</td>
<td>-23.4%</td>
</tr>
</tbody>
</table>

Panel B: Elasticity of Model Parameters

<table>
<thead>
<tr>
<th></th>
<th>Fixed Cost ($f$)</th>
<th>Persistence ($\rho_i$)</th>
<th>Dispersion ($\sigma_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{X}(z_a = H)$</td>
<td>-0.682</td>
<td>-0.988</td>
</tr>
<tr>
<td></td>
<td>$\bar{X}(z_a = L)$</td>
<td>-0.765</td>
<td>-0.807</td>
</tr>
<tr>
<td></td>
<td>$\bar{X}(z_a = H)$</td>
<td>-0.005</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>$\bar{X}(z_a = L)$</td>
<td>0.012</td>
<td>0.398</td>
</tr>
<tr>
<td></td>
<td>Freq ($x &gt; 0, H$)</td>
<td>-0.829</td>
<td>-1.093</td>
</tr>
<tr>
<td></td>
<td>Freq ($x &gt; 0, L$)</td>
<td>-0.916</td>
<td>-0.969</td>
</tr>
<tr>
<td></td>
<td>$E(x \mid x &gt; 0)$</td>
<td>0.376</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>$E(x \mid x &lt; 0)$</td>
<td>0.250</td>
<td>-0.369</td>
</tr>
</tbody>
</table>

for transaction costs (both financial and real), the fixed costs studied here can also be characterized as a search cost that captures the ease of finding trading partners, as in Rhode-Kropf and Robinson (2008). As such, the model suggests that industries with more compatible technology, less specialized assets, and more homogeneous firms (in terms of size) are associated with greater asset purchases and sales.

Finally, although not reported here, the attributes of aggregate productivity ($\rho_a$, $\sigma_a$) have a moderate effect on new investments, but very little impact on asset purchases and sales. Higher persistence ($\rho_a$) makes the current economic state more likely to linger, and higher dispersion ($\sigma_a$) expands the difference between expansions and recessions, both of which lead to higher investments in expansions and lower investments in recessions.
Table V
Summary Statistics: Persistence, Dispersion, and Asset Sales
This table reports summary statistics for different persistence and dispersion groups. The groups are formed such that the high group consists of observations in the top 33 percentile and the low group consists of observations in the bottom 33 percentile based on persistence and dispersion, respectively. For each year, the rate of asset sales is measured as the proportion of plants that change ownership. p-values are reported in parentheses.

<table>
<thead>
<tr>
<th></th>
<th># of Obs</th>
<th>Mean ($\rho$)</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Persistence ($\rho$) and Asset Sales</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-persistence</td>
<td>38</td>
<td>0.56</td>
<td>4.22%</td>
<td>0.90%</td>
</tr>
<tr>
<td>High-persistence</td>
<td>37</td>
<td>0.70</td>
<td>3.80%</td>
<td>0.83%</td>
</tr>
<tr>
<td>t-test</td>
<td></td>
<td>2.11</td>
<td>(3.81)</td>
<td></td>
</tr>
<tr>
<td>Signed rank test</td>
<td></td>
<td>1.62</td>
<td>(10%)</td>
<td></td>
</tr>
<tr>
<td>Spearman correlation</td>
<td></td>
<td>−0.20</td>
<td>(8%)</td>
<td></td>
</tr>
<tr>
<td>Kendall’s tau</td>
<td></td>
<td>−0.15</td>
<td>(6%)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Dispersion ($\sigma$) and Asset Sales</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-dispersion</td>
<td>38</td>
<td>0.15</td>
<td>3.67%</td>
<td>0.78%</td>
</tr>
<tr>
<td>High-dispersion</td>
<td>37</td>
<td>0.30</td>
<td>4.01%</td>
<td>0.88%</td>
</tr>
<tr>
<td>t-test</td>
<td></td>
<td>−1.78</td>
<td>(8%)</td>
<td></td>
</tr>
<tr>
<td>Signed rank test</td>
<td></td>
<td>−1.66</td>
<td>(10%)</td>
<td></td>
</tr>
<tr>
<td>Spearman correlation</td>
<td></td>
<td>0.24</td>
<td>(4%)</td>
<td></td>
</tr>
<tr>
<td>Kendall’s tau</td>
<td></td>
<td>0.15</td>
<td>(5%)</td>
<td></td>
</tr>
</tbody>
</table>

B. Data and Methodology

I use the same sample as in the calibration to test the model’s predictions regarding the relationship between productivity shocks and industry asset reallocation. The sample covers all manufacturing plants from LRD with segment sales greater than $1 million in the real value of shipments in 1982 dollars in the period of 1972 to 2000.

B.1. Productivity Measures

The productivity calculation follows Schoar (2002). I obtain plant productivity from estimating a log-linear Cobb–Douglas production function for each industry-year with the following specification:

$$\ln(Y_{ijt}) = a_{jt} + b_{jt} \ln(K_{ijt}) + c_{jt} \ln(L_{ijt}) + d_{jt} \ln(M_{ijt}) + z_{ijt},$$  \hspace{1cm} (19)

where $Y_{ijt}$, $K_{ijt}$, $L_{ijt}$, and $M_{ijt}$ represent the total value of shipments, the value of capital stock, the equivalent total production man-hours, and value of inputs of plants $i$ in industry $j$ at time $t$, respectively. I calculate the value for capital stock ($K$) as the sum of book value of structures and equipment, and adjust labor inputs ($L$) to reflect both production and nonproduction man-hours.
The Real Determinants of Asset Sales

Inputs \((M)\) are expenses for parts and intermediate goods, fuel, and energy purchased.\(^{28}\)

As pointed out by Schoar (2002), since coefficients on capital, labor, and material inputs may vary by industry and by year, this specification allows for different factor intensities in different industry-years. Moreover, since the constant term \((a_{j,t})\) varies with time, it helps to filter out changes in industry-level productivity. This is very crucial for my study since I want to estimate the persistence and dispersion of firm-level productivity only.

The firm-level productivity is, then, the estimated residual from these regressions, which measures the difference between the actual output and predicted output, given the amount of inputs the plant uses and the mean industry production technology in place. A plant that produces more than the predicted amount of the output has greater-than-average productivity.

### B.2. Persistence and Dispersion

To estimate persistence and dispersion of firm-level productivity, I fit an AR(1) model,

\[
\hat{z}_{ij,t} = \rho_j \hat{z}_{ij,t-1} + \varepsilon_{ijt},
\]

where \(\hat{z}_{ij,t}\) is the estimated firm productivity of plant \(i\) in industry \(j\) at time \(t\) using (19).

I delete industries with less than 50 plants in any given year and industries with less than 5 years of observations in the sample to obtain a stable time series. My final sample consists of 112 industries based on three-digit SIC codes. I use the autoregressive coefficient \(\rho_j\) as my persistence measure, and the standard deviation of the error terms as my measure for dispersion such that \(\sigma_j = \text{std}((\varepsilon_{ijt})_{i=1,t=1}^{n_j,T})\), where \(n_j\) is the total number of plants in industry \(j\) and \(T\) is the total number of years in the sample.

### B.3. Rates of Asset Sales

I define the rate of asset sales as the percentage of plants that change ownership in the industry, similar to Maksimovic and Phillips (2001).\(^{29}\) On average, 3.7% of large manufacturing plants change ownership every year during the sample period. The number is higher in expansion years, approaching 4.3%.\(^{30}\)

\(^{28}\) We adjust for depreciation using the depreciation estimates published by the Bureau of Economic Analysis based on two-digit SIC codes, and use capital expenditures (in real 1982 dollars) spent in the previous year as additions to the capital stock.

\(^{29}\) Results are qualitatively the same when using rates based on the percentage of book value of capital or sales.

\(^{30}\) Using the LRD data from 1974 to 1992, Maksimovic and Phillips (2001) document similar facts. They find an average asset sales rate of 3.89% and a rate of 6.89% for expansion years.
Table VI
Persistence, Dispersion, and Asset Sales: Mean Regressions

This table reports regression results based on (20). All variables are measured as an industry average over the sample period. The rate of asset sales measures the proportion of plants that change ownership. CAPX is measured as capital expenditure divided by lagged assets. ATTURN is measured as total sales divided by total assets. OPMARG is the ratio of earnings divided by sales. ECON measures the percentage of years the industry is in expansion. I define a year to be an expansion year if it has positive sales growth rates for the last 2 consecutive years. $t$-statistics are reported in parentheses and $^*$ and $^{**}$ represent significance at the 10% and 1% level, respectively.

Dependent Variable: Average Rate of Asset Sales (in percentage)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock persistence ($\rho$)</td>
<td>$-3.26^{***}$</td>
<td>$-2.31^{***}$</td>
<td>$-2.01$</td>
</tr>
<tr>
<td></td>
<td>($-2.94$)</td>
<td>($-2.01$)</td>
<td></td>
</tr>
<tr>
<td>Shock dispersion ($\sigma$)</td>
<td>$3.69^{***}$</td>
<td>$2.91^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>($3.30$)</td>
<td>($2.49$)</td>
<td></td>
</tr>
<tr>
<td>CAPX</td>
<td>$-2.50$</td>
<td>$-7.80^*$</td>
<td>$-5.77^*$</td>
</tr>
<tr>
<td></td>
<td>($-0.62$)</td>
<td>($-1.89$)</td>
<td>($-1.38$)</td>
</tr>
<tr>
<td>ATTURN</td>
<td>$-0.55^{***}$</td>
<td>$-0.47^{***}$</td>
<td>$-0.51^{***}$</td>
</tr>
<tr>
<td></td>
<td>($-2.93$)</td>
<td>($-2.55$)</td>
<td>($-2.78$)</td>
</tr>
<tr>
<td>OPMARG</td>
<td>$4.25^{***}$</td>
<td>$3.91^{***}$</td>
<td>$4.23^{***}$</td>
</tr>
<tr>
<td></td>
<td>($3.94$)</td>
<td>($3.70$)</td>
<td>($4.02$)</td>
</tr>
<tr>
<td>ECON</td>
<td>$0.60^*$</td>
<td>$0.49$</td>
<td>$0.55$</td>
</tr>
<tr>
<td></td>
<td>($1.53$)</td>
<td>($1.27$)</td>
<td>($1.43$)</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>$6.86^{***}$</td>
<td>$4.82^{***}$</td>
<td>$6.13^{***}$</td>
</tr>
<tr>
<td></td>
<td>($8.16$)</td>
<td>($8.21$)</td>
<td>($7.03$)</td>
</tr>
<tr>
<td># of Obs.</td>
<td>112</td>
<td>112</td>
<td>112</td>
</tr>
<tr>
<td>Adjusted-$R^2$</td>
<td>25%</td>
<td>26%</td>
<td>28%</td>
</tr>
</tbody>
</table>

C. Tests and Results

I first divide industries in the sample into three groups, each with an equal number of observations, based on persistence $\rho$ and dispersion $\sigma$, respectively. The high persistence group (the top one-third) has an average persistence of 0.70, as compared to 0.56 in the low persistence group (the bottom one-third); and the average dispersion in high and low dispersion groups is 0.15 and 0.30, respectively.

The average amount of asset reallocation differs significantly between high and low persistence groups (3.80% vs. 4.22%), and between high and low dispersion groups (4.01% vs. 3.67%). All differences are significant at the 10% level under both the $T$-test and signed rank test.

Table VI reports test results in a multivariate setting based on the following specification:

$$\bar{s}_j = \beta_0 + \beta_1 \rho_j + \beta_2 \sigma_j + \gamma \cdot \overline{X}_j + \epsilon_j,$$

(21)
The Real Determinants of Asset Sales

where $\bar{s}_j$ is the average rate of asset sales in industry $j$; $\rho_j$ and $\sigma_j$ are persistence and dispersion estimated for industry $j$, respectively; and $X_j$ includes a set of control variables based on industry characteristics.

Both persistence and dispersion are significant factors explaining industry activity, individually or jointly. Industries with less persistent and more dispersed productivity, on average, have greater asset reallocation ($\beta_1 < 0$ and $\beta_2 > 0$). In addition, lower asset turnover and a higher profit margin also lead to more asset sales. Capital expenditure is negatively correlated with asset sales, suggesting that internal growth (capital expenditure) and external growth (asset purchases) may serve as substitutes.

There is evidence that asset reallocation activity can fluctuate considerably over time, even within the same industry. To filter out the time-variant factors that may affect asset sales, I perform a two-step estimation. First, I use the panel data to estimate a model with industry fixed effects, controlling for other characteristics. Then, using the estimated industry fixed effects, I examine whether the variation can be explained by attributes such as persistence and dispersion.

I use the following specification for the first step:

$$s_{jt} = \alpha_j + \beta \cdot X_{j,t-1} + \varepsilon_{jt}, \quad (22)$$

where $s_{jt}$ is the rate of asset sales of industry $j$ from time $t-1$ to $t$ and $X_{j,t-1}$ includes all lagged control variables for that industry. My control variables include capital expenditure (CAPX), asset turnover rate (ATTURN), profitability (OPMARG), demand shock (D.ECON), size distribution (HERF), and the percentage of diversified firms in the industry (MPCT). The results are summarized in Table VII, Panel A. First, asset sales are higher in years when new investments are low and capital utilization rates are low. This is consistent with the model’s prediction that existing assets become substitutes for new assets when excess capacity exists in the industry. The idea that asset reallocation plays a “contractionary” role is also studied in Andrade and Stafford (2004). Second, similar to Maksimovic and Phillips (2001), I find that more assets are traded in expansion years, when the industry experiences positive sales growth. The procyclicality of asset sales can be due to higher efficiency gains or lower financing costs. Industry structures, measured by the Herfindahl index, also matter: More asset sales occur when the Herfindahl index is lower or firms become more homogeneous in size. Profitability, on the other hand, is not a significant factor, and neither is the percentage of diversified firms in the industry. The $F$ test that examines the group effects rejects the null hypothesis that no significant fixed effect exists across industries.

32 CAPX is a ratio of capital expenditure over the lagged assets. ATTURN is measured as total sales divided by total assets. OPMARG is the ratio of earnings divided by sales. D.ECON is a dummy variable that equals one if industry sales increase for the last 2 consecutive years, minus one if sales decrease for the last 2 consecutive years, and zero otherwise. HERF is the Herfindahl Index based on firms’ assets. MPCT is the percentage of plants in the industry that are owned by multisegment firms.
Table VII
Persistence, Dispersion, and Industry Fixed Effects of Asset Sales

This table reports the regression results based on (22) and (23). Panel A shows estimation results of a fixed effects model in which the dependent variable is the annual rate of asset sales in an industry. For each year, the rate of asset sales is measured as the proportion of plants that change ownership. CAPX is measured as capital expenditure divided by lagged assets. ATTURN is measured as total sales divided by total assets. OPMARG is the ratio of earnings divided by sales. D_ECON is a dummy variable that equals one if industry-level sales increase for the last 2 consecutive years, minus one if sales decrease for the last 2 consecutive years, and zero otherwise. HERF is the Herfindahl Index based on total assets. MPCT is the percentage of plants in the industry that are owned by multisegment firms. All explanatory variables are lagged for 1 year. Panel B shows the regression results using the estimated industry fixed effect of asset sales as the dependent variable. In both panels, t-statistics are reported in parentheses and *** represents significance at the 1% level.

### Panel A: Panel Regression

<table>
<thead>
<tr>
<th>Dependent Variable: Asset Sales Rate (in percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPX</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ATTURN</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>D_ECON</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>HERF</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>OPMARG</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>MPCT</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CONSTANT</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R^2$ (overall)</td>
</tr>
<tr>
<td>$F$-stat (all $\alpha_i = 0$)</td>
</tr>
</tbody>
</table>

### Panel B: Industry Fixed Effects

<table>
<thead>
<tr>
<th>Dependent Variable: Industry Fixed-Effects of Asset Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock Persistence ($\rho$)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Shock Dispersion ($\sigma$)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CONS</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td># of Obs.</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>
In the second step, I regress the estimated industry fixed effects $\hat{\alpha}_j$ from (22) on persistence and dispersion:

$$\hat{\alpha}_j = \beta_0 + \beta_1 \rho_j + \beta_2 \sigma_j + \varepsilon_j.$$  \hfill (23)

Panel B reports the results. Industries with lower persistence and higher dispersion have significantly more asset sales ($\beta_1 < 0$ and $\beta_2 > 0$). Together, these two factors explain about 16% of the total variation in the estimated industry fixed effects.

**V. Conclusion**

In this paper, I show that an equilibrium dynamic model of asset sales generates transaction patterns that are consistent with empirical evidence. In my model, asset sales allow firms to optimally adjust their capacity given productivity shocks. By moving resources from less to more productive firms, they improve efficiency in the industry.

The dynamic structure of the model enables me to examine several aspects of asset sales in a very general setting. The main contributions of this paper are as follows.

First, investment decisions are path dependent. I show that changes in productivity brought by shocks affect firms’ decisions to buy or sell assets. The focus on productivity changes rather than levels provides a new perspective to the debate regarding the implication of relative productivity between buyers and sellers.

Second, the model provides new and testable predictions on cross-industry variation in asset reallocation. It suggests that industries with less persistent and more dispersed productivity witness more frequent and bigger productivity changes, and therefore should have greater asset sales. Using the plant-level data from LRD, I find strong support for these predictions.

Third, by identifying the equilibrium asset prices, I am able to model interactions among heterogeneous firms and establish a link between productivity shocks, firms’ investment decisions, and the investment activities in the industry. I show that decisions on asset purchases and sales are not made in isolation. Rather, a firm makes decisions considering its own investment needs as well as the investment needs of all other firms in the industry. New investments are taken when positive aggregate shocks raise the average productivity in the industry, while existing assets are traded when firms’ relative positions change with firm-specific productivity shocks.

Finally, using the simulated panel, I reconcile my model with two empirically documented patterns. In particular, I show that the connection between long-run average valuation and decisions to buy or sell assets can be driven by productivity shocks, rather than misvaluation, and that the negative size effect in acquirer returns can be attributed to the selection problem such that small acquirers demand bigger increases in productivity to take acquisitions.
Appendix A: Proof

**Proposition 1:** There exists a unique function $V(k, z_i; z_a, \overline{k}) : K \times Z_I \times Z_A \times \overline{K} \rightarrow R_+$ that solves the dynamic program in (17) and generates unique optimal policy functions $I(k, z_i; z_a, \overline{k})$ and $x(k, z_i; z_a, \overline{k})$.

**Proof:** Assume that both aggregate and idiosyncratic productivity are bounded such that $z_a \in [z_a, z_a]$ and $z_i \in [z_i, z_i]$.

Since both $z_a$ and $z_i$ follow AR(1) processes, and $0 < \rho_a < 1$ and $0 < \rho_i < 1$, the maximum allowable capital stock $k_{max}$ is determined by $\exp(\overline{z_a} + \overline{z_i})\pi'(k_{max}) - \delta = 0$.

That is,

$$k_{max} = \left[ \exp(\overline{z_a} + \overline{z_i})\alpha \right]^{\frac{1}{\delta}}.$$

It is not economically profitable to have $k > k_{max}$.

Let $K \equiv [0, k_{max}]$.

By definition, $K$ is compact and nonempty. Therefore, the set for the mean of $k, \overline{K}$, is also compact and nonempty, and $\overline{K} \equiv [0, k_{max}]$.

The production function $\pi(.)$ is continuous and concave such that $\pi' > 0$ and $\pi'' < 0$.

The discount rate, $\beta$, is such that $0 < \beta < 1$.

Therefore, the conditions (1)–(3) in Adda and Cooper (2003) are satisfied. By theorem 3 in Adda and Cooper (2003), there exists a unique continuous function $V: K \times Z_I \times Z_A \times \overline{K} \rightarrow R_+$ that solves (17), and there exist stationary policy functions $I(k, z_i; z_a, \overline{k})$ and $x(k, z_i; z_a, \overline{k})$. Q.E.D.

Appendix B: Computation of Approximate Equilibrium

The steps below outline my computation strategy for approximate equilibrium:

- **Step 1:** Guess an initial law of motion $H_0$ with coefficients $\{a_0, a_1, b_0, b_1\}$ and initial pricing function identified as $\{c_0, c_1, d_0, d_1\}$. Solve problem (17) for value function $V(k, z_i; z_a, \overline{k})$, and decision rules $x(k_i, z_i; z_a, \overline{k})$ and $I(k_i, z_i; z_a, \overline{k})$.

- **Step 2:** Use the value function derived from Step 1 and a price grid, $P$, to solve the following problem:

$$\tilde{V}(k_i, z_i; z_a, \overline{k}, P) = \max_{0 \leq I \leq I \times k} \pi(k_i + x, z_i, z_a)
- \left[ I_i + P x_i + \sum_{j \in [k_i, x_i]} \Gamma^j(k_i, j_i) + f x_i \right]_{x_i \neq 0}
+ \beta EV(k'_i, z'_i; z_a, \overline{k}| z_i; z_a, \overline{k}).$$
Assume that the future price will evolve based on the perceived pricing function. Solve for the value function \( \tilde{V}(k_i, z_i; z_a, \bar{k}, P) \) and decision rules \( \tilde{x}(k_i, z_i; z_a, \bar{k}, P) \) and \( \tilde{I}(k_i, z_i; z_a, \bar{k}, P) \).

- **Step 3:**
  (i) Fix the initial capacity/idosyncratic productivity distribution for a large number of firms \( F_0 = \{k^0_i, z^0_i\}_{i=1}^N \), and pick an initial aggregate productivity \( \{z^0_a\} \). Simulate firms’ decisions using the decision rules derived in Step 2. Find the market-clearing price level \( P^0 \).
  (ii) Derive the optimal decisions on asset sales and new investment using the price \( P^0 \) and the decision rules \( \tilde{x}(k_i, z_i; z_a, \bar{k}, P) \) and \( \tilde{I}(k_i, z_i; z_a, \bar{k}, P) \).
  (iii) Calculate the future capacity of all firms and the mean capacity level in the industry: \( k^1_i = (1 - \delta)(k^0_i + x^0_i) + I^0_i \), and \( \bar{k}^1 = \sum_{i=1}^N k^1_i / N \).

- **Step 4:** Generate idiosyncratic and aggregate productivity for the next period using the AR(1) process based on \( (\rho_a, \sigma_a, \rho_i, \sigma_i) \). Repeat a large number of times to obtain a time series of price and mean capacity \( \{P^t, \bar{k}^t\}_{t=1}^T \).
- **Step 5:** Use the stable part of the obtained time series to regress \( \{\log(k_{t+1})\} \) and \( \{P_t\} \) on constants and \( \{\log(\bar{k}_t)\} \) for each value of \( z_a \) to get the realized law of motion \( \hat{H} \) and realized pricing function \( \hat{P} \). Compare the perceived law of motion \( H_0 \) and perceived pricing function \( P^0 \) with the realized law of motion \( \hat{H} \) and realized pricing function \( \hat{P} \). If different, use new coefficients \( (\hat{a}_0, \hat{a}_1, \hat{b}_0, \hat{b}_1, \hat{c}_0, \hat{c}_1, \hat{d}_0, \hat{d}_1) \) to construct the initial guess, return to Step 1, and iterate until convergence.33

**REFERENCES**


33 Convergence is achieved if the norm of the difference in coefficients divided by the norm of the initial coefficients is less than 1%.


