Scramble Teams for the Pinehurst Terrapin Classic

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Michael O. Ball and Russell Halper

Abstract

The Pinehurst Terrapin Classic (PTC) is a five day, 16 player, annual golf tournament that includes match-play and scramble rounds. Pairings must be created for each day of the tournament that ensure each pair of participants play on the same team at least once during the tournament, and ensure that the teams for each scramble round are of comparable ability. Over the past four years, both a simple interactive, spreadsheet-based heuristic and an integer programming model have been developed to create the pairings. These both rely on very special properties of the structure of this tournament. In this paper, we describe this structure, the methods used to create the pairings and the experience with the use of these methods over the most recent years.

KEYWORDS: integer programming, golf pairings, tournaments
1 Introduction

The golf trip: it is an activity that is near-nirvana to the golf aficionado, but its appeal remains a mystery to the non-golfer. A quote from an anonymous non-golfing wife of a golf aficionado: “I fail to see how you could derive enjoyment from getting up at 4:30 in the morning, driving for 6 hours, then playing golf, eating mediocre food and falling into bed dead-tired; then proceeding to get up at 6:00 AM for four straight days, playing golf during all daylight hours and finally coming home completely exhausted”. A quote from Chuck Dotson, a golf aficionado, when asked whether he was available on particular dates for the upcoming Pinehurst Terrapin Classic: “I have no conflict with either date identified. The reason I have no conflict with either date is because I never schedule anything at all for the whole year until I know the EXACT dates for the trip. Then, I schedule dentist and doctors appointments, accept wedding, birthday, bar mitzvah, and christening invitations, attend class reunions, allow myself to have the flu and/or other afflictions and permit my wife to plan mutual trips. I hope this helps.” Thus, it can be seen that the golf trip is an event of extraordinary significance (at least to some) and so it merits precise and meticulous planning, which is the subject matter of this paper.

The Pinehurst Terrapin Classic (PTC) is a prototypical golf trip. It was started well over 30 years ago by the former University of Maryland golf director Frank Cronin. Initially, there were 8 participants and play was held at the famous numbered Pinehurst Resort courses, i.e. Pinehurst Number 1, Pinehurst Number 2, Pinehurst Number 3, etc. Its more recent incarnation involves 16 participants who play at several of the excellent courses in the greater Pinehurst area. Over the years, the participants have included a cross section of members of the University of Maryland community, including vice presidents, deans, faculty and staff of all types and alumni and others who remain close to the University. One attractive aspect of the PTC, and perhaps golf itself, is that it provides a social environment that transcends any organizational hierarchies or boundaries. In fact, there is very little conversation about University business, while the great drive, the great put and the wild shot that careened off a tree and into the pond merit hours of discussion. The PTC takes place over five days starting on Wednesday and ending on Sunday. Play on Wednesday starts around 12 noon with a “standard” 18 hole round. Play on Thursday through Saturday involves an 18 hole standard morning round and an 18 hole afternoon “scramble”. Play finishes on Sunday morning with an 18 hole standard round. The format of these rounds will be discussed in more detail later.

The specific problem we address in this paper is to determine the so-called
“pairings” for the PTC. The pairings are a specification of who plays with whom on a particular day. In this case, 16 golfers participate in the event and golfers play in groups of 4. The pairings are changed each day so the problem is to determine how to partition the 16 golfers into 4 groups on each of the 5 days. There is a social aspect to determining the pairings, namely, we would like each golfer to play with each other golfer, if possible. There is also a competitive aspect, namely, there is a competition set up each day and each group of four represents one team. The groups (teams) should be set up in such a way that the competition is balanced.

It turns out that the specific structure of the PTC (16 golfers, 5 days) is highly desirable and has some remarkable properties. These can be used to readily develop a spreadsheet-based, heuristic approach to addressing the pairings problem. These properties also allow for the formulation of a very compact integer program to solve the problem. This methodology has been used for the past four years to plan for the PTC. Experience with the PTC will be described in detail.

Section 2 of this paper gives background on important golf concepts and defines the problem of interest. Section 3 describes the solution methodology. Experience with the use of this approach in the Pinehurst Terrapin Classic is provided in Section 4.

2 Related Work

Our models for determining pairings for the PTC seek to maximize contact among players and ensure balanced teams. The creation of balanced teams for a single-day scramble tournament was studied by Dear and Drezner [6]. Each team’s ability is measured by a team handicap (i.e. the sum of the handicap of each player in that team). Several metaheuristics are proposed for creating teams that minimize the maximum difference in ability between two teams. Also using team handicap as a measure of team ability, Ragsdale et. al. [12] develop a mixed integer programming model for the creation of balanced teams for a three-day scramble tournament. They additionally consider maximizing contact among players by adding constraints prohibiting any pair of players from being on the same team more than once. Several different objective functions are considered for creating balanced pairings.

Darby-Dowman and Little [4] used integer programming and constraint logic programming to find pairings of four golfers for each day of an 11-day, 12-person match-play tournament to maximize the contact among players. Pairings are created where each player is paired with each other player between
two and four times. No schedule is found where each player is paired with each other player an equal number of times (three) and no consideration is given to creating balanced teams.

Operations research has also been used to examine pairings in professional golf tournaments. Hardy explores strategies for creating Ryder Cup teams ([10], [11]). The focus is on how the team from the United States should schedule its players to compete most effectively against the European team, rather than on how an organizer of a tournament like the PTC should create a fair schedule.

Our models use a combination of a so-called “big hitter index” and a handicap for each player to measure the quality of the members of a scramble team. The incorporation of the big hitter index, which has particular relevance to the nature of a scramble competition, is a concept not found in prior research. Additionally, the format of the PTC induces a very special, perhaps elegant, underlying structure in the pairings that we are able exploit very effectively.

In other settings, tournaments that wish to maximize contact among participants are often scheduled as Round Robin Tournaments ([2], [7], [13]), where each pair of players or teams in the tournament compete against each other at least once in the tournament. Unlike many sports, where two competitors play in each match to determine a winner and loser, the teams created for the PTC are composed of four players and the tournament winner is determined by the players’ collective scores for the tournament rather than the players’ relative performance within each team. Also, as we will see, in some rounds the players on each team cooperate to earn a collective team score. Furthermore, any two participants in a Round Robin Tournament are ensured to have a schedule of roughly the same difficulty since, outside of the match they play against each other, they will face the same opponents. This is not necessarily the case in the PTC. In other tournament settings [9], the location or venue for each round of the tournament is also considered in creating a balanced tournament schedule. It can be desirable, for example, to ensure no team plays all its matches at home or on the road. In the PTC, all teams compete on the same course.

Our problem also resembles some combinatorial problems in timetabling. In [5], de Werra discusses several combinatorial optimization problems on class-teacher timetabling. The author notes a relationship between Latin squares [8] and timetabling, which appears in our work as well. In [1], the author studies the problem of assigning cohorts of students to classes in an MBA program to ensure each pair of cohorts are in the same class at least once during the program. The author points out that this is an application of block scheduling and is similar to the work done in the design of statistical
experiments that was developed by R. A. Fisher. There is no inherent notion of fairness applicable to creating these timetables. Conversely, in [3], fair teams were created using Integer Programming at the Kelly School of Business for projects that undergraduate students must complete to be allowed to take advanced classes. Since, students only participate in one team, there is no notion of maximizing contact among students.

In the next section, we describe models for producing the PTC golf pairings. They use the basic structure described previously, so that social equity is achieved. Further, they insure that the scramble teams are balanced both with respect to the “big hitter” criterion and with respect to total handicaps.

3 Golf Competition Formats, Handicaps and the PTC

Golf has been played for centuries and over that considerable time period has developed a myriad of rules, practices and modes of competing. One development that allows for particularly “fair” competitions to be set up is the handicap system. Each golf hole has a designated par score, which represents what a player should get if he or she plays the hole well. Par on a hole is nearly always 3, 4 or 5. A standard golf course has 18 holes and the sum of the par scores on each of its holes is par for the course. The typical course par score is 72, however, there is usually quite a bit of variation within a very small range, e.g. 70, 71 or 72. Handicaps are computed by keeping a record of a player’s recent scores and computing a deviation from par. More specifically, the 10 best (lowest) scores among the 20 most recent rounds of golf are taken. The average deviation between each of these scores and par is then taken as the player’s handicap. Thus, if a player had a 14 handicap and had only played par-72 courses, then the average of the 10 best scores recorded by that player in the last 20 rounds would be 86. This description has left out a certain component that corrects for the difficulty of a course. This mechanism computes an “effective” par for a course that might be different from the actual par and uses that in the computation (There are a few other technical details related to handicap computations, such as a limit on the number of strokes per hole, that are not relevant here). Note that as a player’s ability improves his or her handicap would decrease. A player with a handicap around 18 is commonly referred to as a bogie golfer since on the average that player scores 1-over-par, a bogie, on each hole. Handicaps are used to create “fair” competitions among players of different abilities. An example of the simple use of handicaps is a competition among several players playing a single round of golf. Each player’s total score would be adjusted by subtracting the player’s
handicap to obtain a net score. The player with the lowest net score would win.

In a standard round of golf an individual player plays all holes on an 18-hole course. A medal play competition would involve individual players playing one or more standard rounds and the winner being determined by lowest total score adjusted by handicap as described above if appropriate. In a match play competition, one or more head-to-head competitions between two individuals are held. Here, total score is not relevant; rather a result is determined for each hole and total holes won or lost determine the match winner. More specifically, a player receives one point if she has a lower score than her opponent on a hole, otherwise no points are received. The player with the most points after 18 (or more) holes wins. Handicaps are applied in the case of match play as follows. The difference, $D$, between the handicaps of the two players is taken. The player with the higher handicap “gets a stroke” on the $D$ most difficult holes on the course, i.e. one stroke is subtracted from the player’s score on the $D$ most difficult holes. If $D$ is greater than 18 then one stroke is subtracted from the higher-handicap player’s score on all holes and a second stroke is subtracted from the $k$ most difficult holes where $k = D - 18$ (the maximum handicap value is 36).

The scramble or captain’s choice golf competition is typically employed in one day golf outings, e.g. corporate or charitable events, that very often involve players of a variety of skill levels. It also is commonly used in the afternoon round when two 18-hole rounds are played in a single day, e.g. as in the PTC. In a scramble, play proceeds on a hole as follows. Each member of a team, usually consisting of 4 players, hits a drive. The “best” of the drives is chosen and one stroke is said to have been taken. Then all players hit from the spot of the chosen drive. The second shot is played in a similar way. That is, the best of the second shots is chosen, a second stroke is recorded and the all players hit from that spot. As soon as one player hits a shot into the hole, the hole is finished and a single score is recorded. While average golfers, e.g. with handicaps between 9 and 18, rarely, if ever, record an 18 hole score at, or under, par, it is not usual for a 4-person scramble team of average golfers to record a score below par. It is most often the case that handicaps are not used to adjust scramble scores in any way, although handicaps are often used in forming scramble teams (as will be done in this paper). The winner of a scramble team competition is usually simply the team with the lowest total score – this is the case for the PTC. There are broad range of slight variations in scramble formats, some quite imaginative, however, these are not relevant for the purposes of this paper. It should be noted that the scramble is not a tournament format officially recognized by golf’s rule-setting bodies – it is
format used for more informal “fun” golf outings.

Let us now return to the specifics of the PTC. As discussed above standard rounds of golf are played on Wednesday, Sunday and the mornings of Thursday, Friday and Saturday. The specifics of the competition used varies from day to day and even year to year. However, the key elements are that all participants play complete, individual 18-hole rounds and handicaps are used to determine the winning individual and team. An example of a method for determining the winning team for a specific day would be to adjust the score of each player on a hole-by-hole basis (as in done in match play as described above) and then to record for each team the sum of the two lowest scores on each hole. Thus, the score for the round would be the sum of the two lowest scores on the 18 holes. While the format can vary significantly, the important feature to note is that in all cases, handicaps are used to make the competition fair. As a result, to a great extent the manner in which pairings are formed is irrelevant. This leaves two remaining objectives to be considered in forming the pairings: 1) the social aspect, i.e. insuring the each participant plays with each other participant at least once and 2) insuring that the afternoon scramble competitions on Thursday, Friday and Saturday are fair.

Over the years, PTC organizers developed a basic approach to forming the pairings. The starting point was to arrange players into four “flights”: A, B, C and D, where the A-players are the four best (as determined by the four lowest handicaps), the B-players are determined by the next four lowest handicaps, etc. On Wednesday, the A players form a team, as do the B players, C players and D players. Since there is only a regular round with handicap adjustments, this competition is fair. Then, on Thursday through Sunday, each team formed consists of one A-player, one B-player, one C-player and one D-player. This structure leads to the formation of balanced scramble teams on Thursday through Saturday since each team contains one player from each skill level. Furthermore, on Wednesday, the players in each flight play together and on each of the subsequent days, a player in one flight plays 4 times with a player in each of the other flights. For example, on Wednesday, a given A-player plays with the other 3 A-players and then on the 4 other days that A-player is on a team with 3 other players (a B-player, a C-player and a D-player). Thus, it is conceivable that a schedule can be formed in which each player plays with every other player exactly one time (we call this property social equity). In fact, a schedule with social equity was found (see Table 1). It was then used over the years by simply designating the appropriate names as player (A,1), (A,2), (A,3), (A,4), (B,1), (B,2), etc.

From time to time some complaints arose relative to the balance of the pairings with respect to the scramble teams. One arises from the rather obvious
### Table 1: PTC Pairings with Social Equity

<table>
<thead>
<tr>
<th>Day</th>
<th>Team#1</th>
<th>Team#2</th>
<th>Team#3</th>
<th>Team#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wednesday</td>
<td>(A,1)</td>
<td>(B,1)</td>
<td>(C,1)</td>
<td>(D,1)</td>
</tr>
<tr>
<td></td>
<td>(A,2)</td>
<td>(B,2)</td>
<td>(C,2)</td>
<td>(D,2)</td>
</tr>
<tr>
<td></td>
<td>(A,3)</td>
<td>(B,3)</td>
<td>(C,3)</td>
<td>(D,3)</td>
</tr>
<tr>
<td></td>
<td>(A,4)</td>
<td>(B,4)</td>
<td>(C,4)</td>
<td>(D,4)</td>
</tr>
<tr>
<td>Thursday</td>
<td>(A,1)</td>
<td>(A,2)</td>
<td>(A,3)</td>
<td>(A,4)</td>
</tr>
<tr>
<td></td>
<td>(B,1)</td>
<td>(B,2)</td>
<td>(B,3)</td>
<td>(B,4)</td>
</tr>
<tr>
<td></td>
<td>(C,1)</td>
<td>(C,2)</td>
<td>(C,3)</td>
<td>(C,4)</td>
</tr>
<tr>
<td></td>
<td>(D,1)</td>
<td>(D,2)</td>
<td>(D,3)</td>
<td>(D,4)</td>
</tr>
<tr>
<td>Friday</td>
<td>(A,1)</td>
<td>(A,2)</td>
<td>(A,3)</td>
<td>(A,4)</td>
</tr>
<tr>
<td></td>
<td>(B,2)</td>
<td>(B,1)</td>
<td>(B,4)</td>
<td>(B,3)</td>
</tr>
<tr>
<td></td>
<td>(C,3)</td>
<td>(C,4)</td>
<td>(C,1)</td>
<td>(C,2)</td>
</tr>
<tr>
<td></td>
<td>(D,4)</td>
<td>(D,3)</td>
<td>(D,2)</td>
<td>(D,1)</td>
</tr>
<tr>
<td>Saturday</td>
<td>(A,1)</td>
<td>(A,2)</td>
<td>(A,3)</td>
<td>(A,4)</td>
</tr>
<tr>
<td></td>
<td>(B,3)</td>
<td>(B,4)</td>
<td>(B,1)</td>
<td>(B,2)</td>
</tr>
<tr>
<td></td>
<td>(C,4)</td>
<td>(C,3)</td>
<td>(C,2)</td>
<td>(C,1)</td>
</tr>
<tr>
<td></td>
<td>(D,2)</td>
<td>(D,1)</td>
<td>(D,4)</td>
<td>(D,3)</td>
</tr>
<tr>
<td>Sunday</td>
<td>(A,1)</td>
<td>(A,2)</td>
<td>(A,3)</td>
<td>(A,4)</td>
</tr>
<tr>
<td></td>
<td>(B,4)</td>
<td>(B,3)</td>
<td>(B,2)</td>
<td>(B,1)</td>
</tr>
<tr>
<td></td>
<td>(C,2)</td>
<td>(C,1)</td>
<td>(C,4)</td>
<td>(C,3)</td>
</tr>
<tr>
<td></td>
<td>(D,3)</td>
<td>(D,4)</td>
<td>(D,1)</td>
<td>(D,2)</td>
</tr>
</tbody>
</table>

The possibility that there could be significant disparity across the A-players (or B-players, etc.). Thus, just insuring each team consists of an A, B, C and D does not necessarily insure balance. For example, for many years the lowest handicap golfer in the group had a handicap above 8. However, recently, Jeff Maynor the Director of Golf at the University of Maryland course joined the group. His handicap is between 0 and 3. This leads to the possibility of somewhat unbalanced teams.

A second, more problematic, issue relates to the number of “big hitters” on a scramble team. There are many styles of golf play, particularly among amateur golfers. Consider two golfers, Chuck and Don, who both have handicaps of 12. Chuck is capable of hitting a drive 250 yards. Thus, he is capable of reaching a long par 4 hole, e.g. 420 yards in length, in two shots and then taking 2 puts to achieve a par. Don, on the other hand, only on the rarest
occasion hits a drive more than 200 yards. His typical drive is say 180 yards. Thus, he would almost never reach a long par 4 in 2 shots. However, Chuck and Don could easily have the same handicap of 12. The reason is that Don is extremely consistent, nearly always hitting the ball straight. Thus, he gets a par or bogie on almost every hole. Chuck, on the other hand, will typically hit 2 or 3 shots very much off line in a typical round. These shots might go into the woods leading to higher scores on those holes, e.g. double or triple bogies (2 or 3 strokes over par). It is important that a scramble team have one or more big hitters. In a scramble a score over par on a hole should be rare. If a team has no big hitters then there will be several par 4 holes in a round when the team will be unable to get on the green in two shots. Even for highly consistent golfers, getting par on all or most such holes becomes a challenge. In addition, par 5 holes should present the scramble team with an opportunity to get a birdie (1 stroke under par). With no big hitters, a scramble team will typically be a relatively long distance, e.g. 100 yards, away from the green after two shots on par 5 holes, making a birdie challenging. In fact, unbalance in this respect has more often led to complaints at the PTC than unbalance relative to overall handicaps as described in the previous paragraph.

In the next section, we describe models for producing the PTC golf pairings. They use the basic structure described previously, so that social equity is achieved. Further, they insure that the scramble teams are balanced both with respect to the “big hitter” criterion and with respect to total handicaps.

4 Modeling the PTC Golf Pairings Problem

Our modeling approach takes as a starting point the basic PTC pairings structure:

1. players are grouped into A, B, C and D flights;
2. on Wednesday, players in the same flight are on the same team;
3. on subsequent days, each group/team consists of a player from each flight.

Let us assume for the moment that we are able to insure social equity. We now consider the issue of evaluating the skill balance of pairings. For reasons discussed above, we take a multi-objective approach by defining two metrics, one based on handicaps and one related to the presence and quality of big hitters on the team. The handicap metric of a team $T$, denoted $Z_{hc}(T)$, is the sum of the handicaps of the players on the team. To obtain the second
metric we start with a subjective evaluation of each player. This subjective evaluation assigns a big hitter index to that player. This index, which varies between 0 and 14, is an estimate of the number of “usable” drives over 200 yards produced by that player in a round of golf. Since a typical golf course has 4 par 3 holes, drives are unnecessary on these holes so the best score is a 14 = 18-4. Here usable means a drive that is in the fairway or short rough so that it would be a likely candidate to be used in a scramble. Some strong golfers might consistently hit drives over 200 yards but a high percentage might go into the woods or other undesirable locations. Other golfers might consistently hit straight drives but only occasionally (or never) hit a drive over 200 yards. These factors would be taken into the subjective evaluation.

Table 2 gives the player list, handicap indexes and big hitter indexes for the 2007 and 2008 PTC participants (there was one change from 2007 to 2008). Also given is the subjective evaluation that went into the determination of the big hitter index. The big hitter index for a team would simply be the sum of the big hitter indices of its players. The team big hitter index is then subtracted from 50 to get a big hitter metric that numerically comparable to the handicap metric. Here by comparable we mean that in both cases lower values are better and both metrics generally fall within the same numeric range. We denote the big hitter metric for a team $T$ as $Z_{bh}(T)$.

Ideal PTC golf pairings would assure that the big hitter metric and the handicap metric are equal for the four teams from each day of the tournament with a scramble round. Since it is likely that such pairings do not exist, we define for a set of PTC golf pairings $P$ an objective function $Z(P)$ that measures the maximum difference in ability between any two teams that compete against each other in a scramble round and attempt to find pairings that minimize its value. A higher score represents less evenly matched, and consequently less desirable, pairings. This objective is calculated as follows. First, a handicap score $Z^d_{hc}$ is computed for each day $d$ of the tournament with a scramble round. This score is the maximum difference in the handicap metric, $Z_{hc}(T)$, over all pairs of teams for day $d$. Secondly, a big hitter score $Z^d_{bh}$ is computed for each day $d$ of the tournament with a scramble round. Similarly, this score is the maximum difference in the handicap metric, $Z_{bh}(T)$, over all pairs of teams for day $d$. The score for the schedule is then defined as the maximum over all days $d$ in the tournament with a scramble round of a weighted sum of $Z^d_{hc}$ and $Z^d_{bh}$. Subjective judgement would be used to choose a set of weights.

Formally, we approach the problem of producing the PTC golf pairings as a minimization problem where we attempt to find a set of golf pairings $P$ that minimizes the objective function $Z(P)$ and satisfy the constraints imposed by the basic PTC pairings structure outlined above. However, since this is
Table 2: Player Input Data

<table>
<thead>
<tr>
<th>Name</th>
<th>Handicap Index</th>
<th>Scouting Report</th>
<th>Big Hitter Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAYNOR</td>
<td>1 2</td>
<td>He is the University of Maryland golf pro – not much more need be said.</td>
<td>14 14</td>
</tr>
<tr>
<td>ORCUTT</td>
<td>8 NA</td>
<td>Outstanding all-around game; has pro length, but can be counted on to hit 1 or 2 in the woods per round.</td>
<td>12 NA</td>
</tr>
<tr>
<td>KRONSER</td>
<td>NA 7.8</td>
<td>Good, consistent all-around game; hits with pretty good length.</td>
<td>NA 12</td>
</tr>
<tr>
<td>HIGGINS</td>
<td>10.2 11.8</td>
<td>Solid all-around game; simple swing produces straight shots with moderate length</td>
<td>6 6</td>
</tr>
<tr>
<td>BALL</td>
<td>11 11</td>
<td>Good all-around game; can hit the ball a long way; mental lapses and a balky putter prevent greatness.</td>
<td>11 11</td>
</tr>
<tr>
<td>RUTHERFORD</td>
<td>11.2 13.9</td>
<td>Has one of the ugliest swings you will ever see but somehow he gets the job done and can hit it a long way.</td>
<td>9 9</td>
</tr>
<tr>
<td>SKERPON</td>
<td>11.3 12</td>
<td>Gets good distance with good-looking swing; suffers with consistency.</td>
<td>8 8</td>
</tr>
<tr>
<td>DOTSON</td>
<td>12 11.9</td>
<td>At one point in time the best golfer in the group; has lost some of his distance and sharpness but still a major threat.</td>
<td>9 9</td>
</tr>
<tr>
<td>EDWARDS</td>
<td>12.1 12</td>
<td>Hits the ball straight with a simple swing; limited body turn reduces distance.</td>
<td>5 5</td>
</tr>
<tr>
<td>FRANKS</td>
<td>12.2 9.8</td>
<td>The quintessential short hitter; great putter; always down the middle but never over 200 yards.</td>
<td>0 0</td>
</tr>
<tr>
<td>TYLER</td>
<td>12.4 13.6</td>
<td>Unbeatable from 50 yards in; hits a very low ball, which limits distance especially in wet or lush conditions.</td>
<td>0 0</td>
</tr>
<tr>
<td>NIXON</td>
<td>12.6 15.1</td>
<td>Hits the ball a long way and, at times, pulls off spectacular shots; inconsistency and chipping woes bring scores up.</td>
<td>10 9</td>
</tr>
<tr>
<td>KESSLER</td>
<td>12.8 17</td>
<td>Hits a draw – which can get out of control when times get tough; otherwise consistent, reliable player.</td>
<td>6 6</td>
</tr>
<tr>
<td>WRENN</td>
<td>13.8 13</td>
<td>Reliable player with great short game; occasionally can crank one out there.</td>
<td>2 2</td>
</tr>
<tr>
<td>GOEBELER</td>
<td>13.9 14.5</td>
<td>Good looking swing that produces distance on drives; frequent mental lapses produce unfortunate shots.</td>
<td>8 8</td>
</tr>
<tr>
<td>COOGAN</td>
<td>19 17.2</td>
<td>Short straight hitter and great chipper.</td>
<td>0 0</td>
</tr>
<tr>
<td>NELSON</td>
<td>20.6 21.8</td>
<td>He has the highest handicap in the group – but a few times per round all moving parts get in synch and he hits a long, straight one.</td>
<td>2 2</td>
</tr>
</tbody>
</table>

a multi-objective problem and since some input data (the big hitter indexes) are subjectively set, we take an approach that allows for some human input in determining a final solution. In remaining parts of this section, we discuss how we determine the golf pairings for the PTC. Firstly, we describe the structure that the PTC pairings structure induces on the schedule. We then describe two approaches that we developed to find balanced PTC golf pairings.
4.1 Equivalence Class of Schedules

We approach the PTC pairings problem by imposing social equity as a constraint and then by searching among all pairings with social equity guided by the metrics defined above. The PTC pairings structure induces a special relation among all pairings with social equity that we exploit when finding a schedule. Let $\mathcal{P}$ be the collection of all PTC golf pairings with social equity. Notice that given PTC golf pairings $P_1$, new pairings $P'_1$ can be generated by switching the position in $P_1$ of any two players of the same flight for every day of the tournament. Note that such an exchange maintains social equity. Similarly, new PTC golf pairings $P''_1$ can be generated from $P_1$ by swapping the four teams from one day with the four teams from a second day, for any two days other than Wednesday. Again social equity is maintained. We call these actions, respectively, type 1 and type 2 swaps. Examining which PTC golf pairings can be generated from a given initial PTC golf pairings $P_1$ reveals an important structure underlying the problem. Define two PTC golf pairings $P_1$ and $P_2$ to be equivalent if PTC golf pairings $P_2$ can be generated from PTC golf pairings $P_1$ through a series of type 1 and type 2 swaps. This defines an equivalence relation $\sim$ on the set $\mathcal{P}$. Let $\mathcal{P} = \mathcal{P}/\sim$ be the set of equivalence classes of PTC golf pairings with social equity. It turns out that $\mathcal{P}$ has a rather strong property that is quite useful in searching through the set of pairings with social equity:

**Theorem 4.1.** $\mathcal{P}$ contains precisely one equivalence class of PTC golf pairings with social equity. Furthermore, given two distinct PTC golf pairings with social equity: $P_1$ and $P_2$, new PTC golf pairings $P'_1$ can be found from $P_1$ through a series of type 1 swaps such that $Z(P'_1) = Z(P_2)$.

The proof of this theorem is in the appendix. This theorem states that any golf pairings for the PTC with social equity can be obtained from any other PTC golf pairings with social equity through a series of the type 1 and type 2 swaps. Furthermore, “optimal” pairings for the PTC can be found through a series of type 1 swaps only.

Probably the most intuitive way to look at this result is to consider a “keystone” generic pairings with social equity, i.e. the pairings given in Figure 1. To use this generic pairings we would merely assign real players to the various generic players listed, e.g. Maynor becomes (A,1), Rutherford, become (A,2), ..., Wren becomes (B,1), etc. The theorem implies that all possible pairings with social equity can be obtained by modifying this assignment, e.g. Rutherford becomes (A,1), Maynor becomes (A,2), ..., Ball becomes (B,1), etc. Thus, in a certain sense the pairings given in Figure 1 is the unique generic pairings with social equity.
4.2 Spreadsheet Based Approach Using Exchanges

Our spreadsheet-based approach to creating a good pairings starts by inputting the generic pairings given in Figure 1. The list of players is also input with their relevant handicaps and big hitter indices. The players are numbered 1 through 16. A correspondence between actual players and the players in the generic schedule is created by associating player indices with the generic player identifiers in the first day’s pairings. Indirect addressing is used to carry over this correspondence to the pairings for the remaining days. Figure 1, Part A illustrates a portion of the spreadsheet. The column heading G refers to a
generic player identifier and the heading A to an actual player number. Note that the generic numbers are repeated in each row with the first row being the A-players, the 2nd row the B-players, etc. Thus, player number 2 is assigned to generic player (A,1), player 1 is generic player (A,2), player 5 is generic player (B,1), etc. The correspondence between actual players and generic players is manually input into the upper left-hand portion of the spreadsheet. The other correspondences are created using indirect addressing. The team metrics given earlier are calculated and the daily metrics, $Z_{hc}^d$ and $Z_{bh}^d$, computed for each day $d$. Weights $w_{hc}^d$ and $w_{bh}^d$ are input and the weighted metric sum $w_{hc}^d Z_{hc}^d + w_{bh}^d Z_{bh}^d$ calculated for each day $d$.

Alternate pairings can be considered by changing the assignment of player indexes to generic player identifiers in the first day’s pairings. The results of the prior section show that all possible pairings with social equity can be generated in this way. As a practical matter, the easiest way to adjust pairing is to interchange identifiers between a pair of generic golfers of the same categories. Thus, the schedule search process would involve carrying out a sequence of such “swaps”. Figure 1, Part B shows the result of a swap. Specifically, players 6 and 8 have been interchanged between generic assignments B2 and B4. This interchange would be manually implemented within the upper left hand portion of the spreadsheet. The changes in the correspondences in the other 3 sub-tables would be calculated automatically using indirect addressing.

Recall that since there is not scramble on Sunday, the Sunday metric values are irrelevant (handicap adjustments insure fairness). This flexibility can be taken advantage of as follows. The spreadsheet automatically generates the weighted sum $w_{hc}^d Z_{hc}^d + w_{bh}^d Z_{bh}^d$ for all days $d$. For any given assignment, the day with the highest weighted sum value can be designated as Sunday and so, a metric for the assignment would be the maximum of the three lowest weighted sum values. Of course, one would want to look at all three weighted sums and even individual team values in evaluating a particular solution. In this way, starting with any initial assignment, swaps could be implemented to “improve” the solution, with each solution being readily evaluated using the metrics displayed. As will be discussed in the implementation section, this process was used to generate multiple candidate solutions, which were then subjectively evaluated.

### 4.3 Integer Programming Based Approach

Theorem 4.1 allows us to compactly model the problem of determining pairings for the PTC as an integer program (see [14] for background). Similar to the spreadsheet, a feasible solution to the integer program is an assignment of the
actual players to the player identifiers in the generic schedule in Table 1. The objective of the integer program is to minimize the value of $Z(P)$, as defined in Section 4. Keeping this in mind, we now formulate the integer program. For notational convenience in our formulation, rather than referring to each player by their player numbers, we index the players from each flight by the numbers one through four. In other words, MAYNOR would be golfer 1 of flight A, ORCUTT would be golfer 2 of flight A, RUTHERFORD would be golfer 1 of flight B, etc.

Let $D$ be the set of days in the PTC with a scramble round (i.e. Thursday, Friday, and Saturday). Let $F = \{A, B, C, D\}$ be the collection of flights. Let $hc_{(f,i)}$ be the handicap and $bh_{(f,i)}$ be the big hitter index of golfer $i$ from flight $f$. For each day $d \in D$, let $T_d^j \subset F \times \{1, 2, 3, 4\}$ be the collection of generic player identifiers in Table 1 of team $j$ from day $d$. For each $f \in F$, golfer $i \in \{1, 2, 3, 4\}$ of flight $f$, and $s \in \{1, 2, 3, 4\}$, we define the binary variable $X_{f,s,i}^j$ to be 1 if golfer $i$ of flight $f$ is to be assigned to the generic player identifier $(f, s)$ and 0 otherwise. Additionally, for each day $d \in D$, we define the variable $Z_{hc}^d$ and $Z_{bh}^d$ to be, respectively, the maximum and minimum handicap, and $\overline{Z}_{hc}^d$ and $\underline{Z}_{bh}^d$ to be the maximum and minimum big hitter index of any team on day $d \in D$. Our integer program for finding a fair PTC golf pairing is as follows:

\[(IP) \min Z \] 
\[\text{subject to: } Z \geq w_{hc}Z_{hc}^d + w_{bh}Z_{bh}^d \text{ for } d \in D \] 
\[\sum_{s=1}^{4} X_{f,s,i}^j = 1 \text{ for } f \in F, i = 1, 2, 3, 4 \] 
\[\sum_{i=1}^{4} X_{f,s,i}^j = 1 \text{ for } f \in F, s = 1, 2, 3, 4 \] 
\[Z_{hc}^d = \overline{Z}_{hc}^d - Z_{hc}^d \] 
\[Z_{bh}^d = \overline{Z}_{bh}^d - Z_{bh}^d \] 
\[\overline{Z}_{hc}^d \geq \max_{1 \leq j \leq 4} \sum_{(f,s) \in T_d^j} \sum_{i=1}^{4} hc_{(f,i)}X_{s,i}^f \text{ for } d \in D \] 
\[\underline{Z}_{hc}^d \leq \min_{1 \leq j \leq 4} \sum_{(f,s) \in T_d^j} \sum_{i=1}^{4} hc_{(f,i)}X_{s,i}^f \text{ for } d \in D \]
\[
Z_d^{bh} \geq \max_{1 \leq j \leq 4} \sum_{(f,s) \in T_d^j} \sum_{i=1}^4 bh(f,i)X_{s,i}^f \quad \text{for } d \in D \quad (9)
\]

\[
Z_d^{bh} \leq \min_{1 \leq j \leq 4} \sum_{(f,s) \in T_d^j} \sum_{i=1}^4 bh(f,i)X_{s,i}^f \quad \text{for } d \in D \quad (10)
\]

\[
X_{s,i}^f \in \{0, 1\} \quad \text{for } f \in F; s = 1, 2, 3, 4 \quad i = 1, 2, 3, 4 \quad (11)
\]

\[
Z, Z_{hc}, Z_{bh}, Z_{hc}^d, Z_{bh}^d, Z_{bh}^d \geq 0 \quad \text{for } d \in D. \quad (12)
\]

Above, constraints (3) and (4) ensure that for each flight, each golfer is assigned to a single, distinct generic player identifier. Constraints (5) and (6) determine the maximum difference between two teams in the handicap and big hitter metrics for each day \(d \in D\). Constraints (7)-(10) determine for each day \(d \in D\), the maximum and minimum handicap and big hitter metric for a team. Unlike the spreadsheet, the integer program above need only define constraints for the days of the tournament with a scramble round \((D = \{2, 3, 4\})\). This may be done as a result of Theorem 4.1, which states that pairings \(P\) that minimize \(Z(P)\) through type 1 swaps alone. Constraint (2) then allows the IP to minimize the maximum disparity in team abilities over all days \(D\), the days with a scramble round. Consequently, for any feasible assignment, the objective function in this IP is equal to \(Z(P)\) for the pairings dictated by that assignment.

5 Application and Experience

The spreadsheet-based approach was first used in 2005 to generate the PTC golf pairings. The integer program was applied to the 2007 data (after the tournament was over) and then used in 2008 in concert with the spreadsheet approach to generate pairing for use in the PTC. The player input data for 2007 and 2008 are given in Table 2. We note that this is a multi-objective problem; further some subjectivity is used in generating the input data. Thus, it would not seem to be appropriate to try to set parameter values, generate an “optimal” solution and put it into use. In 2007, using the approach described in Section 4.2, four different candidate solutions were generated. All numeric information (handicap values, big-hitter values, etc.) was stripped away from the resultant pairings and then they were evaluated by the 4-person PTC planning committee. This committee came to a collective agreement on the “best” of the four solutions. This is perhaps an example of a classic approach to multi-objective optimization where automation is used to generate a very
small set of candidate solutions and subjective judgement is used to make a final choice. In this particular case, the four solutions considered had the characteristics given in Table 3.

<table>
<thead>
<tr>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Max-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.2</td>
<td>10.6</td>
<td>11.1</td>
<td>9.36</td>
<td>11.1</td>
</tr>
<tr>
<td>9</td>
<td>14.3</td>
<td>11.9</td>
<td>16.1</td>
<td>14.3</td>
</tr>
<tr>
<td>7.36</td>
<td>20.8</td>
<td>4.84</td>
<td>11.4</td>
<td>11.4</td>
</tr>
<tr>
<td>11.6</td>
<td>11.8</td>
<td>13.4</td>
<td>11.6</td>
<td>11.8</td>
</tr>
</tbody>
</table>

Table 3: 2007: Maximum Weighted Metric Differences for Each Solution Generated Using Spreadsheet Approach

The values in the table are differences between the weighted sum of metric values for the team with the highest value and the team with the lowest value. These are given for the pairings on each of the 4 days. As discussed earlier, the “worst” daily pairings is assigned to Sunday so that an overall metric is given by the max of the three lowest values. This is given in the fifth column. The committee chose the solution associated with row 1. It is interesting to note that the committee chose the solution with the lowest value of this metric. One might argue that the solution given in the third row is more desirable in that it has the two “best” days and its third best day is only marginally worse than the one in the solution chosen (row 1).

It certainly can be argued that the validity of the overall process could only be determined based on several years of experience since the outcome in any given year is highly uncertain depending on the performance level of individuals on a given day. It is instructive to review the 2007 outcomes to get some indication of the competitiveness of the schedule. On Thursday, two team tied for first place in the scramble with a score of -5 (5 under par) and two teams tied for second with a score of -4. On Friday, the best score was -8 and the worst -3. On Saturday, there was 3-way tie for first place at -3 with the fourth team at -1. By any criterion the matches on two days (Thursday and Saturday) were extremely competitive and on the third (Friday) less so. This outcome would seem to provide some (anecdotal) evidence of the effectiveness of this approach.

The integer program above was implemented in Express-IVE and ran on a Dell Latitude D630 with 2GB of memory. In our implementation, we reduced the number of variables by assuming that player $i$ in flight $A$ is assigned to the generic player identifier $(A, i)$ for flight $A$ for each day of the tournament. Since the players in flight $A$ must appear somewhere in the schedule, this assumption...
is nonrestrictive and serves to reduce the number of variables in the model. The integer program executed in less than 1 second. The results are shown in Table 4. The schedule found by the integer program has a significantly lower value of the metric than any schedule found using the spreadsheet exchange method. It fact we found this somewhat surprising as we had assumed that the (manual) swapping heuristic would generate near-optimal solutions. The substantial improvement in solution quality gives some indication of the usefulness of integer programming in this setting. We should also point out that our own subjective evaluation of the schedule was that it was indeed significantly better than the others.

<table>
<thead>
<tr>
<th></th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.32</td>
<td>6.68</td>
<td>5.64</td>
<td>6.68</td>
</tr>
</tbody>
</table>

Table 4: Maximum Weighted Metric Differences for the Optimal Solution to the Integer Program

In 2008, the integer program was used to generate a solution for possible use in the PTC. Two additional solutions were generated using the spreadsheet-based approach. These three solutions, which are summarized in Table 5, were submitted to the PTC planning committee (without quantitative information). Note that the third solution is the one generated by the integer program (of course it has the lowest – optimal – maximum deviation).

<table>
<thead>
<tr>
<th></th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Max-3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.6</td>
<td>10.2</td>
<td>10.2</td>
<td>24</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>9.7</td>
<td>3.6</td>
<td>12.8</td>
<td>24.8</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td>7.7</td>
<td>8.9</td>
<td>8.8</td>
<td>25.3</td>
<td>8.9</td>
</tr>
</tbody>
</table>

Table 5: 2008: Solutions Submitted to Committee (two generated by spreadsheet approach and one generated by integer program)

The committee chose the third (IP based) solution. The choice would seem to provide some indication of the validity of the modeling approach used. The scramble rounds were less competitive in 2008 than in 2007. On Thursday the winning score was -10 and the 4th place was -2. On Friday, two teams tied for first with -9 and the 4th place score was -2. On Saturday the first place score was -6 and 4th place -1. Note that a slightly different scramble format was used, which may have led to the overall lower scores.
6 Conclusions

The results of this paper show that the format used by the PTC has a very rich and perhaps intriguing structure. This structure allows us to achieve a tournament which has excellent social structure (each player is able to play one day with each other player). Moreover, this structure allows us to efficiently find a highly competitive set of pairings. While this approach has been implemented for only a short time, implementation experience to date is very positive.

Our approach took advantage of very particular characteristics of the PTC: 5 days, 16 players. Thus, our techniques cannot be applied directly to tournaments with other formats. However, certain key concepts we developed certainly can be used to address these other cases, e.g. the notion of social equity and the big hitter index. Most likely more complex integer programs (or other solution methods) would be required.

At a more general level, our research adds to the growing evidence of the richness of sports scheduling problems and the ability of operations research to provide effective tools for their solution.

APPENDIX: Proof of Schedule Equivalence Theorem

We now present the proof of Theorem 4.1. The proof shows that under certain, non-restrictive assumptions, the schedule has a unique representation. We begin by making a few observations about feasible PTC pairings.

Observation 1 By design, each team on Wednesday is composed of all players of a flight. Consequently, on the subsequent days of the tournament \((d = 2, 3, 4, 5)\), each team must consist of exactly one generic player identifier from each flight.

Observation 2 We may assume for every day, the generic player identifier \((A,i)\) is in team \(i\) for \(i = 1, 2, 3, 4\).

Observation 3 By the equivalence relation \(\sim\), we may assume that the generic players identifiers in each teams on Thursday are as follows.

Observation 4 As a consequence of Observation 3, we may assume that on Friday, Saturday, and Sunday, each team must contain one generic player identifier of each index.
Observation 5  Since PTC golf pairings remain equivalent under type 1 swaps, one may assume that team 1 on Friday consists of the generic player identifiers \((A, 1), (B, 2), (C, 3), \) and \((D, 4)\).

Since by Observation 3 we may assume that the generic player identifiers in each team on Wednesday and Thursday are uniquely determined, we will focus on how the composition of the teams on Friday affects the pairings for the remainder of the tournament.

Lemma 6.1. **Suppose the generic player identifiers in each team on the first three days of the tournament are defined. Assuming feasible pairings exists, the generic player identifiers in each team on Saturday and Sunday are uniquely determined up to switching of the pairings on those two days.**

*Proof.* Consider scheduling team 1 of day 4. As noted in Observation 1, each team in the schedule must contain one generic player identifier from each flight \(f\) (for \(f = A, B, C, D\)) and one generic player identifier of each index \(i\) (for \(i = 1, 2, 3, 4\)). Noting Observation 2, suppose the pairings from the third day are as follows.

<table>
<thead>
<tr>
<th>Group#1</th>
<th>Group#2</th>
<th>Group#3</th>
<th>Group#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A,1)</td>
<td>(A,2)</td>
<td>(A,3)</td>
<td>(A,4)</td>
</tr>
<tr>
<td>(B,i_{1,B})</td>
<td>(B,i_{2,B})</td>
<td>(B,i_{3,B})</td>
<td>(B,i_{4,B})</td>
</tr>
<tr>
<td>(C,i_{1,C})</td>
<td>(C,i_{2,C})</td>
<td>(C,i_{3,C})</td>
<td>(C,i_{4,C})</td>
</tr>
<tr>
<td>(D,i_{1,D})</td>
<td>(D,i_{2,D})</td>
<td>(D,i_{3,D})</td>
<td>(D,i_{4,D})</td>
</tr>
</tbody>
</table>

Observation 5 states that \(i_{1,B} = 2, i_{1,C} = 3, \) and \(i_{1,D} = 4\). Then there are precisely two choices of index, \(i\) and \(j\), for which flight B generic player identifier that may be in team 1 on Saturday, the fourth day of the tournament. In particular, one may write \(\{i, j\} = \{2, 3, 4\}\backslash\{i_{1,B}\}\). Suppose the generic player identifier \((B,i)\) is in team 1 on Saturday. By Observation 4, either the generic player identifier \((C,j)\) or \((D,j)\) must then be in team 1 on Saturday. This choice is uniquely dictated as, again by Observation 4, either \(i_{1,C} = j\) or \(i_{1,D} = j\). The final generic player identifier in team 1 must then be either...
We denote the set of generic player identifiers in this first team on Saturday as \(\{(A, 1), (B, j_B), (C, j_C), (D, j_D)\}\).

Now choose another team \(t (t = 2, 3, 4)\) from Saturday and consider the player of flight \(f (f = B, C, D)\). There are at most two generic player identifiers from a flight that can be in team \(t\). In particular, a player of flight \(f\) in team \(t\) must have an index from the set, numerically, \(\{1, 2, 3, 4\}\) \(\{t, i_t, f, j_f\}\). Furthermore, since \(i_t, f \neq t\) and \(j_f \neq t\), the choice of index is unique if \(i_t, f \neq j_f\). This equality may hold for at most three times. Consequently, six of the nine, remaining generic player identifiers on Saturday are uniquely determined. Once those generic player identifiers are considered, at least two of the remaining three generic player identifier must be uniquely determined, after which the final generic player identifier may be assigned. Consequently, the schedule for Saturday is uniquely determined after assigning player \((B, j_B)\) to team 1 on that day. On Sunday, every generic player identifier has been on a team with all but three other generic player identifiers. Thus the Sunday pairings are uniquely determined given the pairings from the other four days of the tournament.

The previous lemma establishes that given the above assumptions, the pairings on Friday uniquely determine the pairings for the remaining two days, up to swapping the two days. The next step is to determine which pairings on Friday allow for feasible schedules. Some additional notation must first be defined. Let \(\sigma_{d, t, f}\) be the index of the generic player identifier of flight \(f\) from team \(t\) on day \(d\). For example, the pairings in Table 1 dictates \(\sigma_{3, 1, B} = 2\). Choose a day \(d\) after Thursday. Then we look at the permutations \(\sigma_f\) for \(f = B, C, D\), written in matrix form,

\[
\sigma_f = \begin{bmatrix}
\sigma_{d, 1, A} & \sigma_{d, 2, A} & \sigma_{d, 3, A} & \sigma_{d, 4, A} \\
\sigma_{d, 1, f} & \sigma_{d, 2, f} & \sigma_{d, 3, f} & \sigma_{d, 4, f}
\end{bmatrix}.
\]

**Lemma 6.2.** For any day \(d\) after Thursday, \(\sigma_f\) may not be a 4-cycle for any \(f\) in any feasible pairings.

**Proof.** Suppose, without loss of generality, at \(\sigma_2\) is a 4-cycle. Then one may write

\[
\sigma_2 = \begin{bmatrix}
s_1 & s_2 & s_3 & s_4 \\
s_2 & s_3 & s_4 & s_1
\end{bmatrix}.
\]

Two possible pairings then exist for the remaining part of the day. Temporarily ignoring the assumptions made in observation 2, the first pairings are: The second pairings are similar to the first pairings, except that the
indices from flights C and D are swapped in each team. As a result we present the proof for only the first set of pairings. There are then two possible teams that may contain \((A, s_1)\) on Saturday. The team will either be \(\{(A, s_1), (B, s_3), (C, s_4), (D, s_2)\}\) or \(\{(A, s_1), (B, s_4), (C, s_2), (D, s_3)\}\). In either case, two players who played together on Friday, also play together on Saturday, violating the PTC pairings pairings structure. Thus \(\sigma_f\) may not be a 4-cycle.

A corollary of the last theorem is that \(\sigma_f\) must be a permutation consisting of two 2-cycles. Notice that there are exactly 3 of those permutations:

\[
\pi_1 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix},
\]

\[
\pi_2 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix},
\]

and

\[
\pi_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}.
\]

Thus if on day \(d\), a generic player identifier of index \(i\) and flight \(f\) is in team 1, then the second row of permutation \(\pi_{i+1}\) determines the order that the indices for that flight will appear in the table of pairings. Consequently, by Observation 5, we may assume the pairings on Friday must have the generic player identifiers as in Table 1. Then, by lemma 6.1, the schedule must be equivalent to the schedule in Table 1. Thus all schedules are equivalent under type 1 and type 2 swaps.

To prove the second claim of the theorem, it suffices to show that a sequence of type 1 swaps exists that move the teams from any chosen three of the final four days of the tournament in Table 1 to Thursday, Friday, and Saturday (in no particular order), and moves the teams from the remaining day to Sunday. To do so, determine the teams that will be on Sunday and choose a a set of type 1 swaps that switches that set of teams with the teams on Sunday in Table 1. Notice that since a type 1 swap only moves players within a flight,
the schedule on Monday is preserved. Similar to the proof of Lemma 6.1, the teams on the other two days must be uniquely determined. The teams on those two other days may be swapped between the two days, but remain otherwise the same. Thus, an optimal schedule may be found through type 1 swaps only. This proves Theorem 4.1.

References


