Measuring and Understanding Uncertainty of Uncertainty

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Abstract

Uncertainty of uncertainty characterizes the dispersion of the (percentage change to the) cost of insuring equities. We propose a methodology that measures and extracts uncertainty of uncertainty from options on the VIX futures. Uncertainty of uncertainty is high, variable, and not highly correlated with extant uncertainty indexes. Exploring its macroeconomic origins in the setting of a large macroeconomic data set, we find that uncertainty of uncertainty can be forecast by principal components that echo concerns about monetary policy outcomes, flight to safety, and deflation. We draw inferences about the predictive coefficients based on a number of statistical tests, including a parametric bootstrap procedure.

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1. Introduction

The theme of this paper is to exploit the information in the options market and formalize the notion of uncertainty of uncertainty, and then investigate its link to macroeconomic primitives. We propose a static positioning in call and put options on the VIX futures to develop an analytical representation for the dispersion of VIX futures returns.

Since dispersion of VIX futures returns is quantified from options with VIX futures as the underlier, we term our new measure uncertainty of uncertainty. Crucial to our approach, one can gain exposure to uncertainty through VIX futures and options, as the VIX is not directly traded. Uncertainty of uncertainty is a uniquely important economic object to study and characterize, as it provides an estimate of the dispersion of the (percentage change to the) cost of insuring equities.

There are three attributes of our approach. First, we develop a formula that captures uncertainty of uncertainty. This formula is parsimonious in the observables, namely, the prices of VIX calls and puts, as a function of strike prices, and is model-free. In particular, our methodology does not depend on distributional assumptions about the dynamics of VIX futures.

Next, the estimates of uncertainty of uncertainty are consistent with the underlying natural probability distribution of uncertainty and the risk aversion of investors (and are in the flavor of the “wisdom of the crowds”). Testifying to the feature that the VIX options market is active and vibrant, the total daily call (put) volume is 722,643 (186,930), and the open interest is 3,511,737 (1,253,239), contracts on March 17, 2016. Finally, we show that uncertainty of uncertainty is analytically distinct from (i) the integrated variance and (ii) the tail size, of VIX futures returns.

Our work uncovers salient properties of uncertainty of uncertainty over the expiration cycles
of the VIX options. For one, the size of uncertainty of uncertainty is high (95% annualized on average) and fluctuates substantially with a minimum of 48.9% and a maximum of 172.2%. Hinting to the unique information content of uncertainty of uncertainty, we show that its correlation with the uncertainty indexes of Jurado, Ludvigson, and Ng (2015) and Baker, Bloom, and Davis (2015) is $-0.25$ and $0.34$, respectively. Our evidence further indicates that uncertainty of uncertainty has low correlations with the distress index of the Federal Reserve Bank of St. Louis and the VIX.

Motivated by the above findings, we ask: What is the source of variation in uncertainty of uncertainty? What type of information is encapsulated within uncertainty of uncertainty?

To investigate these questions, we study the predictive link between the principal components extracted from macroeconomic primitives (i.e., the large data panel of McCraken and Ng (2015)), and uncertainty of uncertainty. The empirical finding is that uncertainty of uncertainty can be forecast, in a statistically significant manner, by principal components that we show are related to concerns about monetary policy outcomes, flight to safety, and deflation, with an $R^2$ of 25.4%.

While we rely on a number of ways to draw inferences, our conclusions about predictability hinge on a parametric bootstrap procedure. The design of the bootstrap is aligned with the data feature that some principal components obey a low-order ARMA($p$, $q$) model, with a negative moving average component. Our simulation results indicate that a parametric bootstrap (under the said data features and persistent predictors) can alleviate size distortions in small samples.

Thus, we go further in showing individual and joint parameter significance, and in safeguarding the robustness of conclusions. In so doing, we respect the essence of the approaches and exercises in Amihud, Hurvich, and Wang (2009), Ludvigson and Ng (2009), Bakshi, Panayotov,


Differing in their focus and analytical constructions, Huang and Shaliastovich (2014) show that the vvix index of the CBOE helps to characterize the cross-section and time series of S&P 500 index and VIX option returns, whereas Song (2014) considers VIX options to show that volatility-of-volatility and jump-induced volatility tails are priced. The big picture in our paper is that uncertainty of uncertainty is variable and high. Moreover, we present a model-free estimator of uncertainty of uncertainty, and we study the macroeconomic origins of uncertainty of uncertainty via predictive regressions. Our approach is rooted in established econometrics, and enables novel and reliable insights about the structure of uncertainty of uncertainty. The center piece of our study is to measure and understand uncertainty of uncertainty.

2. Measuring uncertainty of uncertainty

This section achieves three objectives. First, it formalizes our concept of uncertainty of uncertainty. Second, it presents a new model-free approach that quantifies uncertainty of uncertainty. Third, it provides a theoretical and conceptual distinction of uncertainty of uncertainty from the
integrated variance of VIX futures returns and the tail size of VIX futures returns.

2.1. Quantifying uncertainty of uncertainty

Our reliance is on options whose underlier are VIX futures. We recognize in our work that VIX measures the cost of protecting equities (see footnote 1) and that VIX is not directly traded.

Let $F_t = \mathbb{E}_t^* (\text{VIX}_{t+1})$ be the level of the VIX futures at date $t$, and let $F_{t+1}$ be the corresponding level at date $t + 1$. Throughout, $\mathbb{E}_t^* (\cdot)$ represents expectation under the risk-neutral measure (that is consistent with the underlying natural probability measure, discounting, as well as the risk aversion of investors; see, for example, Singleton (2006, equation 8.23)).

Pertinent to our theoretical and empirical rationale, the percentage changes in VIX futures price exhibit a correlation of 0.88 with the percentage changes in the VIX. Hence, we take percentage changes in VIX futures to surrogate movements in VIX uncertainty.

Define the random variable

$$z_{t+1} \equiv \frac{1}{F_t} (F_{t+1} - F_t), \quad (1)$$

which captures the (excess) return of a fully collateralized long futures position. The long futures position is profitable when the cost of insuring equities rises over $t$ and $t + 1$. One could also interpret $z_{t+1}$ as a normalized uncertainty shock (under the risk-neutral measure), since $F_{t+1} - \mathbb{E}_t^* (F_{t+1})$ equals $F_{t+1} - F_t$ by the martingale property of the futures price, i.e., $\mathbb{E}_t^* (z_{t+1}) = 0$.

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1The VIX futures and the VIX can be complicated objects to model parametrically, where VIX$^2$ is the price of the 30-day variance swap, approximated by a portfolio of calls and puts on the S&P 500 index (e.g., Carr and Lee (2008, Sections 3 and 4), Sepp (2008, Sections 4 and 5), Carr and Wu (2009), Cont and Kokholm (2013), Amengual and Xiu (2013, Section 2), Mencía and Sentana (2013, Section 4), Aït-Sahalia, Karaman, and Mancini (2015), Mixon and Onur (2015), Song and Xiu (2016), and Dew-Becker, Giglio, Le, and Rodriguez (2016)).
To articulate a theoretical measure of uncertainty of uncertainty, we consider the construct of Jensen’s gap, denoted by $D(z_{t+1})$, as

$$D(z_{t+1}) \equiv \mathbb{E}^*_t(g(z_{t+1})) - g(\mathbb{E}^*_t(z_{t+1})) \geq 0, \text{ for a convex function } g(z_{t+1}).$$  \hspace{1cm} (2)

Set the function $g(z_{t+1}) = z_{t+1}^2$, which is strongly convex (as $g''(z_{t+1}) = 2 > 0$ for all $z_{t+1}$). Then

$$D(z_{t+1}) = \mathbb{E}^*_t(z_{t+1}^2) - (\mathbb{E}^*_t(z_{t+1}))^2 = \mathbb{E}^*_t(z_{t+1}^2).$$  \hspace{1cm} (3)

The design of the $z_{t+1}$ is both intuitive and crucial, as it circumvents the need to determine the risk-neutral mean of $z_{t+1}$, which is identically zero. Such a conclusion would not follow if, for instance, had we defined $z_{t+1} = \log(F_{t+1}/F_t)$. With the random variable $z_{t+1}$ centered around zero by construction, our analysis indicates that $D(z_{t+1}) = \mathbb{E}^*_t(z_{t+1}^2)$ is a dispersion measure.

Important for the tasks at hand, there is data on options on the VIX futures. In this light, how could one extract $\mathbb{E}^*_t(z_{t+1}^2)$? Exploiting Carr and Madan (2001, equation (1)) and/or Bakshi, Kapadia, and Madan (2003, equation (2)), we have for, any integer $n$, $g(z_{t+1}) = z_{t+1}^n \in C^2$, the space of twice-continuously differentiable functions, with (successive) derivatives $g'(\cdot)$ and $g''(\cdot)$, that

$$g(F_{t+1}) = g(F_t) + g'(F_t)\{F_{t+1} - F_t\}$$

$$+ \int_{K > F_t} g''(K) \max(F_{t+1} - K, 0) dK + \int_{K < F_t} g''(K) \max(K - F_{t+1}, 0) dK.$$  \hspace{1cm} (4)
Since $g(F_t) = 0$ and $E_t^* (F_{t+1}) = F_t$, we, thus, obtain the key analytical expression

$$
E_t^* (z_{t+1}^n) = \int_{K>F_t} \frac{(n)(n-1)R_{t+1}(\frac{K}{F_t}-1)^{n-2}}{F_t^2} C_t[K] dK + \int_{K<F_t} \frac{(n)(n-1)R_{t+1}(\frac{K}{F_t}-1)^{n-2}}{F_t^2} P_t[K] dK,
$$

(5)

where $C_t[K]$ and $P_t[K]$ are the time $t$ prices of the out-of-the-money (OTM) call and put options on the VIX futures as a function of the strike price $K$. The gross return of a (one-period) riskfree bond is $R_{t+1}$. Importantly, for $n = 2$ in equation (5), we express uncertainty of uncertainty as

$$
u_{ou_t} \equiv \sqrt{E_t^* (z_{t+1}^2)}, \quad \text{where } E_t^* (z_{t+1}^2) \text{ is derived via the options portfolio in equation (5).} \quad (6)
$$

Throughout, we will refer to $\nu_{ou_t}$ as uncertainty of uncertainty, and measure it using the information in the prices of European options written on the VIX futures.

Notably, $\nu_{ou_t}$ embeds a structure, whereby each OTM option is weighted equally by $2R_{t+1}/F_t^2$ each $t$. The size of $\nu_{ou_t}$ is varying stochastically, and a higher value conveys richer convexity.

Analogous to the notion of semivariance, we can express upside uncertainty of uncertainty as

$$
\nu_{ou_t}^+ = \sqrt{\int_{K>F_t} \frac{2R_{t+1}}{F_t^2} C_t[K] dK},
$$

(7)

which isolates dispersion on the upside of $z_{t+1}$, i.e., when $\{F_{t+1} > F_t\}$. Empirically, we show that $\nu_{ou_t}^+ / \nu_{ou_t}$ is high. Economically, this feature arises since VIX calls are more expensive than VIX puts and the risk-neutral distribution of VIX futures is positively skewed and fat-tailed.
2.2. Further discussion and comparison with the \( \text{vvix}_t \) index of CBOE

Our measure, \( uou_t \), which is based on Jensen’s gap, is methodologically distinct from the \( \text{vvix}_t \) measure of the CBOE (2014):

\[
\text{vvix}_t^2 = \int_{K > F_t} \frac{2R_{\text{rf},t+1}}{K^2} C_t[K] dK + \int_{K < F_t} \frac{2R_{\text{rf},t+1}}{K^2} \mathbb{P}_t[K] dK.
\] (8)

The above formula stems from assuming that the VIX futures follows \( dF_t/F_t = \sigma_t d\omega_t \) for Brownian motion \( \omega_t \), provided instantaneous volatility \( \sigma_t \) satisfies \( \int_t^{t+1} \sigma_s^2 ds < +\infty \). Thus, the \( \text{vvix}_t^2 \) is calculated in the same fashion as the VIX\(^2 \) is calculated, using the S&P 500 index options, as also implemented in Huang and Shaliastovich (2014, equation (3.2)) and Park (2015, Section 2.2).

The calculation in equation (8) implies that \( \text{vvix}_t^2 \) is the price (at inception) of the swap on the integrated variance \( \int_t^{t+1} \sigma_s^2 ds \). The price can be replicated through a portfolio of VIX options.

One potential drawback of the formula for \( \text{vvix}_t \) adopted by CBOE is that it assigns heavier weights, \( 2R_{\text{rf},t+1}/K^2 \), to OTM puts on VIX futures. The higher weighting for low strike puts is counterintuitive as the risk-neutral distribution of \( z_{t+1} \) exhibits positive asymmetries.

We recognize, guided by an analogy with a result in Carr and Wu (2009, equation (5)) and Du and Kapadia (2012, equation (30)) based on a semimartingale dynamics for equity, that when VIX options assign a value to \( \{uou_t^2 - \text{vvix}_t^2\} \), namely, \( \mathbb{E}_t^* \left( z_{t+1}^2 - \int_t^{t+1} \sigma_s^2 ds \right) \), the positive (negative) discrepancy approximates the price of discontinuous uncertainty moves to the upside (downside).
2.3. Comparison with the tail size of uncertainty

The measure $\text{uou}_t$ is theoretically distinct from the tail size of VIX futures returns. Consider the composition of the function $f(z)$ with $g(z)$ as $h(z) = (f \circ g)(z) \equiv f(g(z))$.

By virtue of a Taylor expansion of $f(g(z))$ around $\mathbb{E}^*(g(z))$, we have that

$$f(g(z)) = f(\mathbb{E}^*(g(z))) + (g(z) - \mathbb{E}^*(g(z))) f'(\mathbb{E}^*(g(z))) + \frac{1}{2} (g(z) - \mathbb{E}^*(g(z)))^2 f''(\mathbb{E}^*(g(z))) + \ldots$$

(9)

Now consider two convex functions $f(z_{t+1}) = z_{t+1}^2$ and $g(z_{t+1}) = z_{t+1}^2$. Based on equation (9), we have $\mathbb{E}^*(f(g(z_{t+1}))) - f(\mathbb{E}^*(g(z_{t+1}))) = \text{Var}^*(z_{t+1}^2) = D(z_{t+1})$, which characterizes the tail size:

$$\text{tail size}_t \equiv \mathbb{E}^*\left((z_{t+1}^2)^2\right) - (\mathbb{E}^* (z_{t+1}^2))^2,$$

(10)

$$= (\mathbb{E}^* (z_{t+1}^2))^2 \left( \frac{\mathbb{E}^* (z_{t+1}^4)}{(\mathbb{E}^* (z_{t+1}^2))^2} - 1 \right),$$

(11)

$$= \text{uou}_t^4 (\text{Kurtosis} - 1). \quad \text{(since } z_{t+1} \text{ is centered around 0)}$$

(12)

To compute tail size$_t$, we need the option positioning underlying $z_{t+1}^d$. We can show that

$$\mathbb{E}^*_t(z_{t+1}^4) = \int_{K > F_t} \frac{12 R_{t+1}^{\text{rf}}}{F_t^2} \left( \frac{K}{F_t} - 1 \right)^2 \mathbb{C}_t[K] dK + \int_{K < F_t} \frac{12 R_{t+1}^{\text{rf}}}{F_t^2} \left( \frac{K}{F_t} - 1 \right)^2 \mathbb{P}_t[K] dK.$$\hspace{1cm} (13)

The tail size of uncertainty maps to uou$_t$ and to the kurtosis of $z_{t+1}$, and is positive by construction.

Finally, the uncertainty of uncertainty measures in (6) and (7) depart conceptually from Agarwal, Arisoy, and Naik (2016, Section 1.3, Table 1). Their measure, denoted by $\text{vov}_t$, is the realized return of lookback straddles on the VIX futures, and is intended to capture a trend-following strat-
egy on the VIX futures. Moreover, our analytical treatment of the dispersion of $z_{t+1}$ is different from Volkert (2014), who has examined the distribution of VIX using non-parametric methods.

3. Deciphering uncertainty of uncertainty

Central to our analysis is the analytical formulation of uncertainty of uncertainty and the availability of a market in VIX options. In this light, we address two fundamental questions. First, what are the empirical properties of $u_{ou_t}$? Second, what are the economic forces underlying variations in $u_{ou_t}$? To answer the latter question, we conduct a forecasting exercise in the setting of a large macroeconomic data set. Large data sets are imperative to the conclusions of Stock and Watson (2002a, 2002b, 2003, 2016), Bernanke, Boivin, and Eliasz (2005), Ludvigson and Ng (2007, 2009), and Jurado, Ludvigson, and Ng (2015).

3.1. Options on the VIX futures

To implement the formula for uncertainty of uncertainty in equation (6), we require the VIX futures price, and OTM option prices on the VIX futures. Our sample of VIX options consists of daily closing bid and ask prices, and we take the midpoint of the bid and ask.

VIX options, which started trading in March 2006, had insufficient volume in the first two months. Hence, we consider the history from May 2006 (5/18/2006) to March 2016 (3/17/2016). This sample contains 119 expiration cycles. The VIX options are of the European style and expire on the Wednesday that is 30 days prior to the third Friday of the calendar month immediately
following the expiring month.

While there is data on VIX futures across all maturities, and across all contracts and expirations on the VIX options, we focus on the nearest-maturity options over monthly expiration cycles and construct the data at the start of each cycle. The days to expiration is between 27 and 35 days.

We adopt a number of filters to construct the call and put option price series. As is conventional, we first exclude in-the-money VIX calls and puts. Second, we exclude observations for which the bid prices are lower than the minimum tick size of $0.1, or the ask is lower than the bid. Finally, we calculate the implied volatility corresponding to the Black (1976) model, and cross-check the integrity of option price observations, especially across adjacent strikes.

The VIX options is the second most active option contract after the S&P 500 index. Table Appendix-I presents a snapshot of the number of strikes for OTM calls and puts, the trading volume, and the open interest, tabulated at the beginning of the expiration cycle. Germane to our calculations, the sample contains a total of 1,425 calls versus 645 puts, with an average of 12.0 calls and 5.4 puts per expiration cycle. The VIX options market is bifurcated with higher open interest and volume on calls than puts.

3.2. Not so tranquil uncertainty of uncertainty

Our characterization of $\omega_t$ in equation (6) bypasses the need to parametrically model VIX futures. More specifically, the time series of $\omega_t$ is extracted based on a portfolio of VIX calls and puts via the trapezoidal rule, at the beginning of the expiration cycles. When necessary, we construct a grid of implied volatilities (analogous to Carr and Wu (2009)) and then use Black’s
model to fill potential gaps across the quoted VIX option strikes.

The estimates of \( \text{uou}_t \) reported in Table 1 correspond to a window of 30 days (on average) and are in annualized percentage units. The average \( \text{uou}_t \) is 94.1% with a standard deviation of 21.8%. The 5th (95th) percentile value is 61.2% (129.7%). The minimum recorded \( \text{uou}_t \) of 48.9% indicates that the level of uncertainty of uncertainty is relatively high. The tail size of VIX futures return, \( \text{tail size}_t \), has an average of 0.054, and is volatile with a standard deviation of 0.048. These salient aspects of \( \text{uou}_t \) deserve reconciliation by the theorists and appreciation by the empiricists.

A visual observation from Figure 1 is that uncertainty of uncertainty is far from tranquil. Moreover, the test of Bai and Perron (1998, 2003) reveals the presence of one structural break on February 22, 2010, and is associated with an increase in the level of \( \text{uou}_t \), from 81% to 106%.

While our sample period has been eventful with the great recession, turmoil in the banking and housing sectors, and unprecedented response of central banks in the form of quantitative easing, the variable and high threshold of uncertainty of uncertainty is still puzzling.

Table 1 also enables a comparison of \( \text{uou}_t \) and \( \text{vvix}_t \), and reveals that \( \text{vvix}_t \) has both a lower average (annualized) value of 89.3% and a lower standard deviation of 14.9%. Figure 1 additionally shows that while the two series comove together (the correlation is 0.88) due to their common dependence on the VIX options, the estimates of \( \text{uou}_t \) often exceed those of \( \text{vvix}_t \).

There are several ways to frame and highlight the documented higher level of uncertainty of uncertainty. First, the \( \text{vvix}_t \) formula is predicated on a diffusion process for the VIX futures and does not incorporate jump effects. In contrast, Bandi and Reno (2016) show that equity price and
its volatility co-jump. Notably, we rely on a generic pricing and spanning characterization that is independent of the form of jump and/or diffusion assumptions for VIX futures. Moreover, our formulation is derived as an exact dispersion measure.

The final distinction is that our formula equally weights OTM VIX calls and puts, whereas \( \text{vvix}_t \) overweights OTM VIX puts. The form of our proposed weighting scheme is appropriate since the risk-neutral distribution of VIX futures is positively skewed.

To render these points precise, we consider the data on 10/23/2014, when the VIX futures is at 17.3 and \( R_{rf,t+1} = 1.00 \). The expiration date is 11/19/2014. Our calculations show that the weights on VIX options that underlie \( \text{vvix}_t \) are declining in \( K \), with 28 day \( \text{uou}_t^2 - \text{vvix}_t^2 \approx 0.00439 \).

<table>
<thead>
<tr>
<th>Strikes, K</th>
<th>OTM VIX Puts</th>
<th>OTM VIX Calls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
<td>14.5</td>
</tr>
<tr>
<td>Option prices, $</td>
<td>0.28</td>
<td>0.40</td>
</tr>
<tr>
<td>( \text{uou}<em>t ) weight (i.e., 100 ( \times \frac{2R</em>{rf,t+1}}{F_t^2} ))</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>( \text{vvix}<em>t ) weight (i.e., 100 ( \times \frac{2R</em>{rf,t+1}}{K^2} ))</td>
<td>1.02</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Complementing the above insight, we also compute \( \text{asymmetry}_t \equiv \frac{\mathbb{E}_t^* (z_{t+1}^3)}{(\mathbb{E}_t^* (z_{t+1}^2))^{3/2}} \), where

\[
\mathbb{E}_t^* (z_{t+1}^3) = \int_{K > F_t} \frac{6R_{rf,t+1}}{F_t^2} \left( \frac{K}{F_t} - 1 \right) C_t[K] dK + \int_{K < F_t} \frac{6R_{rf,t+1}}{F_t^2} \left( \frac{K}{F_t} - 1 \right) P_t[K] dK, \tag{14}
\]

and \( \mathbb{E}_t^* (z_{t+1}^2) \) is displayed in equation (5). The average value of \( \text{asymmetry}_t \) is 2.47 and ranges between 1.18 and 4.32. Hence, the positive skewness of \( z_{t+1} \) is an intrinsic trait of the data.

The variation in \( \text{uou}_t \) is dominated by \( \text{uou}_t^+ \) with a correlation of 0.98. We further note that the
average level of \( uou_t^+ \) is 85.7% with a standard deviation of 20.7%. Moreover, \((uou_t^+)^2\) captures a large fraction of \((uou_t)^2\): the average \((uou_t^+)^2/(uou_t)^2\) is 82.7%, and the ratio varies between 51.6% and 93.7%. Thus, \( uou_t^+ \) is a predominant component of uncertainty of uncertainty.\(^2\)

Finally, how is \( uou_t \) correlated (considering lag, contemporaneous, and lead effects) with extant measures of uncertainty? We explore this question from several perspectives and present our findings in Table 2. For example, \( uou_t \) is reliably negatively correlated \((-0.25)\) with the Jurado, Ludvigson, and Ng (2015) macroeconomic uncertainty index, given that the 95% bootstrap confidence intervals do not straddle zero. Moreover, the correlation with the economic policy uncertainty index of Baker, Bloom, and Davis (2015) is reliably positive \((0.34)\). Next, the contemporaneous correlation of \( uou_t \) with the distress index (Federal Reserve Bank of St. Louis) and the VIX (averaged over the expiration cycles) is \(-0.21\) and 0.08, respectively. Overall, a noteworthy finding is the absence of a large shared covariation between \( uou_t \) and uncertainty indexes. This finding motivates our analysis of \( uou_t \) in the context of a large macroeconomic data set.

### 3.3. Macroeconomic origins of uncertainty of uncertainty

Throughout this subsection, we center attention on a forecasting exercise, where the predictive variables are principal components constructed using the macroeconomic data set of McCraken and Ng (2015). We rely, in particular, on a parametric bootstrap for drawing statistical inference about the predictive regression coefficients, given the econometric concerns outlined in, among

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\(^2\)One can gauge the average magnitude of \( uou_t \) in relation to average VIX futures return. If one equates a long position in the VIX futures to a trade that is intended to hedge the downside of equities (e.g., Cheng (2013) and Branger, Kraftschik, and Volkert (2015)), then such a trade has experienced an average return of -3.6% per month with a minimum (maximum) return of −35% (137.3%). This trade has an annualized standard deviation of 88%. 


### 3.3.1. There is support for 10 principal components from the large macroeconomic data set

As elaborated in McCraken and Ng (2015, Appendix), the data on macroeconomic primitives is drawn from a broad spectrum of the economy and the considered 135 series are divided into eight groups: (1) output and income; (2) labor market; (3) housing; (4) consumption orders, and inventories; (5) money and credit; (6) interest and exchange rates; (7) prices; and (8) stock market.

To construct a balanced panel, we start our sample from January 1978 (the University of Michigan Index of Consumer Expectations is quarterly prior to 1978). Moreover, we confine our analysis to those series that do not have missing observations. The resulting data for the principal component analysis contains 123 macroeconomic series over January 1978 to February 2016.

Prior to implementing the panel information criterion of Bai and Ng (2002) to select the number of principal components, we appropriately transform each data series and standardize them to have zero mean and unit variance. The procedure supports the presence of 10 principal components, \( \{ \text{PC}^{(j)}_t : t = 1, \ldots, T, \text{and } j = 1, \ldots, 10 \} \). These 10 principal components together explain 56.7% of the total variance. In particular, the first five principal components, i.e., \( \text{PC}^{(1)}_t \) through \( \text{PC}^{(5)}_t \), capture the lion’s share of the total variance, which is 42.7%. 
3.3.2. Econometric approach and addressing the nuances about inference

Our econometric approach follows, among others, Stock and Watson (2002a) and Ludvigson and Ng (2009), and focuses on both individual and joint parameter significance of the coefficients in the predictive regressions below:

\[
\text{uo}_t = \beta_0 + \sum_{j=1}^{10} \beta_j \text{PC}_t^{(j)} + \epsilon_t,
\]

(15)

where \(\{\text{PC}_t^{(j)} : t = 1, \ldots, T, j = 1, \ldots, 10\}\) are the estimated principal components, \(\epsilon_t\) is a disturbance term, and \(\text{uo}_t\) is a univariate time-series to be forecast. Our conclusions are robust when \(\log(\text{uo}_t)\) is the variable to be forecast. Therefore, the results with \(\log(\text{uo}_t)\) are omitted.

Considering the combinations of 10 principal components, there are potentially 1,024 predictive regressions underlying the specification in equation (15). We then select predictor combinations, subject to minimized Bayesian information criterion (BIC).

Across the space of 10 principal components, the minimized BIC of 6.0245 is obtained with \(\text{PC}_t^{(2)}, \text{PC}_t^{(3)}, \text{PC}_t^{(8)},\) and \(\text{PC}_t^{(9)}\), as opposed to a BIC of 6.193 with the 10 principal components as predictors. The selection result is robust to the inclusion of higher-order effects. Thus, we feature the regressions with four predictors in Table 3, without incorporating additional squared or cubic terms. Reported are the OLS coefficients, together with the associated two-sided \(p\)-values.

The forecasting literature has been concerned with a number of statistical issues. Predominant among those is the persistence of the predictors (Cochrane (2015, Section 3)). Table Appendix-II shows that the decay of the autocorrelations for some predictors is slow and at odds with what
can be described through autoregressive models. In contrast, the autocorrelation patterns could be compatible with a low order ARMA\((p,q)\) process, that has a negative moving average component.

Information criteria, such as the BIC, suggest that ARMA(1,1) provides a reasonable parametric model for \(PC_t^{(3)}\) and \(PC_t^{(8)}\). For \(PC_t^{(3)}\), the coefficient on the AR(1) component is 0.99 and on the MA(1) component is \(-0.83\), whereas for \(PC_t^{(8)}\) the coefficient on the AR(1) component is 0.97 and on the MA(1) component is \(-0.88\). In contrast, \(PC_t^{(2)}\) and \(PC_t^{(9)}\) are ARMA(0,0).

Whereas near-unit-root behavior by some of the predictors could be an issue given our evidence, the \(p\)-values from the augmented Dickey and Fuller (1979) and Phillips and Perron (1988) tests reject the null of a unit root in each time-series. The highest \(p\)-value is 0.08 for \(PC_t^{(3)}\). Figure 2 plots the time variation in the four predictors.

[Figure 2 about here.]

Wei and Wright (2013) argue that the Hodrick (1992) 1B covariance matrix estimator can provide reliable inference with persistent (autoregressive) predictors, even in small samples. Guided by their arguments, we compute the Hodrick (1992) estimator, in addition to the HAC estimator of Newey and West (1987), where the lags are chosen automatically using Newey and West (1994).

Importantly, we also implement a parametric bootstrap procedure following Amihud, Hurvich, and Wang (2009, Appendix C), Bakshi, Panayotov, and Skoulakis (2011, Appendix C), Bauer and Hamilton (2015, Section 2.3), and Mark (1995, Section III). In particular, the parametric bootstrap is intended to accommodate the negative MA(1) component present in some of the predictors. In addition, the Monte Carlo simulation shows that the parametric bootstrap is closer to delivering the correct size in small samples, details of which are provided in the Appendix. The gist of
our simulation analysis is that the parametric bootstrap can help to guard against issues of small sample and possible disturbance distribution misspecification.

3.3.3. Interpretation of the empirical results and takeaways

We center our analysis on the test of individual and joint significance of the slope coefficients in the following restricted predictive regression:

\[ u_{ou_t} = \beta_0 + \beta_2 PC_{t-1}^{(2)} + \beta_3 PC_{t-1}^{(3)} + \beta_8 PC_{t-1}^{(8)} + \beta_9 PC_{t-1}^{(9)} + \varepsilon_t. \]  

Particular attention is devoted to inference in the presence of persistent predictors with a negative MA component.

A question that is both economically and statistically important (in light of Bauer and Hamilton (2015)) is whether \( u_{ou_t} \) is highly persistent. The best model for \( u_{ou_t} \) is ARMA(1,1), selected according to the BIC, with an AR(1) coefficient of 0.86 (p-value of 0.000) and an MA(1) coefficient of \(-0.45\) (p-value of 0.000). The message is that \( u_{ou_t} \) series is not overly persistent and the nonstationary of \( u_{ou_t} \) is not a concern, as is also evident from Figure 1.

Table 3 presents our predictive regression results with \( PC_{t-1}^{(2)}, PC_{t-1}^{(3)}, PC_{t-1}^{(8)}, \) and \( PC_{t-1}^{(9)} \) as the predictors. The coefficients on the four predictive variables are individually statistically significant according to the Newey and West (1987) HAC standard errors, with optimally selected lag \( \ell^* = 6 \), and the Hodrick (1992, IB) standard errors. The largest p-value is 0.048 on \( \beta_8 \).

There are departures when drawing inference based on the parametric bootstrap. First, the
\( \beta_3 \) estimate is borderline statistically significant with a \( p \)-value of 0.097. Second, the \( p \)-value on \( \beta_8 \) rises to 0.067. The \( \beta_9 \) estimate has a \( p \)-value of 0.000 under all three methods for drawing inference. As seen, the parametric bootstrap furnishes the most conservative \( p \)-values. The joint \( p \)-value with the parametric bootstrap is 0.012 (the joint \( p \)-values remain lower than 0.05 in the regressions with two and three predictors).

It is of economic interest to ask what fraction of the variation in \( u_{out} \) is explained by the principal components. In this regard, the regression with four predictive variables has an adjusted \( R^2 \) of 25.4%. The explanatory ability remains strong at 21.2% when \( \text{PC}_{t-1}^{(3)} \) is omitted.

How robust are our findings? Instead of lagging the principal components once in the predictive regressions, we lag them twice and present the findings in Table 4. The inclusion of an additional lag mitigates potential concerns about the proximity of the start of the VIX option expiration cycle to the public release of some of the macroeconomic data. Our assessment shows that the sign of the coefficient estimates \( \beta_2, \beta_8, \) and \( \beta_9 \) remains intact, and their statistical significance is preserved. However, as would be expected, the goodness-of-fit adjusted-\( R^2 \) declines to 18.1%.

While Tables 3 and 4 suggest that some of the principal components help to forecast \( u_{out} \) based on the \( p \)-values from the parametric bootstrap, what is the economic interpretation of these findings? In particular, what features of the economy underpin our predictability results?

[Figure 3 about here.]

[Figure 4 about here.]

Table 5 and Figure 3 present the \( R^2 \), when the macroeconomic primitives are regressed on
each of the principal components, i.e., PC\(_t^{(2)}\), PC\(_t^{(3)}\), PC\(_t^{(8)}\), and PC\(_t^{(9)}\), in a univariate regression. For parsimony, we focus on 15 macroeconomic primitives with the highest \(R^2\). To gauge the sign of the relation, we also report the associated pairwise correlations in Table 5 and Figure 4.

PC\(_t^{(2)}\) has the footprint of inflation/deflation. It is negatively and highly correlated with \(\Delta^2\) log of CPI and personal consumption expenditure, both of which are identified by Stock and Watson (2003) as inflation variables (as is conventional, \(\Delta\) indicates the first difference of a series). An additional (unreported) exercise reveals a correlation of \(-0.38\), \(-0.30\), and \(-0.27\) between PC\(_t^{(2)}\) and the yield of a one-, 10-, and 30-year Treasury bond, reinforcing our view that PC\(_t^{(2)}\) embodies changes in inflation expectations. Table 3 notes that \(\beta_2 > 0\), imparting the interpretation that deflation concerns are particularly important to equity investors.

It is further seen from Table 5 that inflation and housing effects are commingled within PC\(_t^{(3)}\). While PC\(_t^{(3)}\) shares high positive correlation with movements in the inflation variables, the correlation with the log of the housing variables (particularly, housing starts and new private housing permits) is negative and of similar magnitude. Given the estimated \(\beta_3 > 0\), our interpretation is that \(\beta_3\) largely reflects concerns about the economic health of the housing sector.

The interpretation of PC\(_t^{(8)}\) is multifaceted. Of the 15 variables with the highest \(R^2\), five of them are drawn from group 6 (interest and exchange rates) where the U.S. dollar is the reference against other currencies, and are positively correlated with PC\(_t^{(8)}\). For example, PC\(_t^{(8)}\) exhibits a correlation of 0.61 (0.56) with percentage change in the trade weighted U.S. dollar index (Switzerland/U.S. exchange rate). Since the U.S. dollar is a safe asset and appreciates during bad economic times, we deduce that \(\beta_8\) represents sensitivity to concerns about flight to safety.
We beef up our interpretation from another angle. Specifically, change in the average duration of unemployment has a negative correlation of −0.45 with \(PC_t^{(8)}\). The change in the average duration of unemployment largely emanates from employed people experiencing job losses. Notably, during the initial phase of economic downturns, a larger portion of workers are laid off (e.g., Valletta (2013)). Considering that a person with long unemployment history is less likely to obtain a new job compared with those who recently lost their jobs, a decrease of the duration of unemployment coincides with bad economic times.

Collecting the evidence about the economic meaning of \(PC_t^{(8)}\) (including a negative correlation of −0.39 between \(PC_t^{(8)}\) and \(\Delta\log\) of new orders for durable goods), we argue that \(\beta_8\) manifests concerns about flight to safety amid deterioration of economic conditions and is consistent with some of the evidence in Baele, Bekaert, Inghelbrecht, and Wei (2013, Table 7).

There may be some ambivalence in how to interpret \(PC_{t-1}^{(9)}\) as it positively predicts \(\text{uo}_t\), and yet is positively correlated with (i) \(\Delta^2\log\) M2 money stock, (ii) \(\Delta\log\) of real personal income (excluding transfer receipts), (iii) \(\Delta\log\) of civilian employment, and (iv) \(\Delta\) of all employees, financial activities (employees on nonfarm payrolls engaged in financial activities). Thus, improving economic indicators are associated with higher levels of \(PC_{t-1}^{(9)}\), which raises subsequent \(\text{uo}_t\). In this light, we interpret \(\beta_9\) as the sensitivity to concerns about monetary policy outcomes. Specifically, improving indicators are viewed adversely by the financial markets, as it hinders Federal Reserve Board policies on monetary easing and Treasury bond buybacks. Said differently, an improving economy raises doubts about the sustainability of a low interest rate environment, which increases the dispersion of the percentage change to the cost of protecting equities. The magnitude of the predictive coefficient \(\beta_9\) is the highest, and is uniformly significant in Tables 3 and 4.
While we have emphasized interpretations based on a dynamic factor model approach that exploits cross sectional information embodied in the principal components, we complement our analysis by asking one final question. Specifically, does the evidence from predictive regressions with individual macroeconomic primitives conform with our interpretations?

We consider the following algorithm to achieve dimension reduction:

1. Regress $u_{t}$ on each lagged macroeconomic variable that is maximally correlated with $PC_{t-1}^{(2)}$, $PC_{t-1}^{(3)}$, $PC_{t-1}^{(8)}$, and $PC_{t-1}^{(9)}$ (as enumerated in Table 5). We winnow our analysis to 18 predictors with $p$-values below 0.1, according to Newey and West (1987, 1994).

2. Select the set of predictors that minimize the BIC in multivariate predictive regressions.

The results reported in Table 6 uncover a finding that four macroeconomic primitives are quantitatively important for forecasting $u_{t}$. Among the four variables, (i) log housing starts-midwest, and (ii) $\Delta$log Switzerland/U.S. exchange rate, are highly correlated with $PC_{t}^{(3)}$ and $PC_{t}^{(8)}$, respectively, while (iii) $\Delta$log real personal income excluding transfer receipts, and (iv) $\Delta$log all employees, financial activities, are highly correlated with $PC_{t}^{(9)}$. Thus, our interpretations based on Table 5 are consistent with the sign and parameter significance highlighted in Table 6. The results additionally illustrate, for example, that a one-standard deviation appreciation (i.e., 2.6%) of the U.S. dollar against the Swiss Franc tends to increase $u_{t}$, on average, by 4.79.

The takeaways from Tables 3, 4, 5, and 6 are several folds. First, our BIC-based selection criterion shows that a few principal components are sufficient to forecast uncertainty of uncertainty. Our findings appear robust when the predictors are lagged twice. Next, we feature a parametric bootstrap procedure to conduct inference when some of the predictors have a negative MA com-
ponent. Importantly, our bootstrap analysis shows that principal components related to concerns about monetary policy outcomes, flight to safety during bad times, and deflation are drivers of uncertainty of uncertainty in a regression framework, exhibiting individual and joint parameter significance. The results with individual macroeconomic primitives reinforce our findings.

4. Concluding remarks

The uncertainty of uncertainty characterizes the dispersion of the (percentage change to the) cost of insuring equities. Our innovation is to propose a methodology that measures and extracts uncertainty of uncertainty from options on the VIX futures. Uncertainty of uncertainty is high and variable in the considered sample period of May 2006 to March 2016 (over expiration cycles).

Our empirical work raises and addresses the key question of identifying the sources of variation in uncertainty of uncertainty. We explore this question in the setting of predictive regressions and dynamic factor models where the principal components are distilled from a large macroeconomic data set, much like Stock and Watson (2002a, 2002b, 2003), Ludvigson and Ng (2009), Jurado, Ludvigson, and Ng (2015), and others, have done to understand inflation, output, and term premiums of Treasury bonds.

Our findings are promising in the sense that four principal components explain 25.4% of the future variation in uncertainty of uncertainty. Our interpretation is that these principal components are associated with concerns about monetary policy outcomes, flight to safety during bad times, and deflation. In particular, our conclusions about individual and joint parameter significance are robust under different methods for constructing standard errors, and we specifically feature a
parametric bootstrap that yields a significantly improved size in small samples, and in the presence of negative (and pronounced) moving average component in some predictors.

While we have taken steps to parse the nature of the embedded information, there is room for developing and testing theories about how uncertainty of uncertainty evolves through time. This agenda is complementary to research that explores a role for uncertainty shocks, for example, Christiano, Motto, and Rostagno (2014), Bianchi, Ilut, and Schneider (2015), Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2015), Di Tella (2015), and Orlik and Veldkamp (2014).
Appendix: Monte Carlo simulation

We conduct a Monte Carlo simulation to show that our proposed parametric bootstrap procedure provides significantly improved size (in small samples) in the presence of a negative moving average component in some of the predictors.

Evidence (e.g., Wei and Wright (2013) and Bollerslev, Marrone, Xu, and Zhou (2015)) suggests that the Newey and West (1987) procedure may inflate statistical significance in small samples when the predictors are strongly persistent. The intuition is that the \( t \)-statistics underlying the predictive coefficients may not be adequately approximated by the standard normal distribution.

To be concrete, we consider the bivariate predictive regression as an illustration:

\[
\text{ou}_{t} = \beta_0 + \beta_8 \text{PC}_{t-1}^{(8)} + \beta_9 \text{PC}_{t-1}^{(9)} + \epsilon_t, \tag{A1}
\]

where \( \text{ou}_t \) is the series to be forecast and \( \text{PC}_{t-1}^{(8)} \) and \( \text{PC}_{t-1}^{(9)} \) are the predictors (the principal components). Under the null of no predictability, we have \( \beta_8 = \beta_9 = 0 \).

The simulation entails the following dynamics:

\[
\begin{align*}
\text{ARMA}(1,1) : & \quad \text{ou}_t = 0.1295 + 0.86 \text{ou}_{t-1} - 0.45 \tilde{\eta}_{t-1,u} + \tilde{\eta}_{t,u}, \quad \tag{A2} \\
\text{ARMA}(1,1) : & \quad \text{PC}_{t}^{(8)} = 0.97 \text{PC}_{t-1}^{(8)} - 0.88 \tilde{\eta}_{t-1,pc8} + \tilde{\eta}_{t,pc8}, \quad \tag{A3} \\
\text{ARMA}(0,0) : & \quad \text{PC}_{t}^{(9)} = \tilde{\eta}_{t,pc9} \quad \tag{A4} \\
(\tilde{\eta}_{t,u}, \tilde{\eta}_{t,pc8}, \tilde{\eta}_{t,pc9}) & \text{are each } N(0,1) \text{ and independent.} \quad \tag{A5}
\end{align*}
\]
The model selection is based on the BIC, and the model parameters (each are statistically significant) are estimated using the actual data on $u_{0t}$ and the principal components. The estimation method is conditional least squares.

In the simulation exercise, we fix the number of time-series observations to be 119 (as our data on $u_{0t}$). Additionally, we do 50,000 simulation trials, $m = 1, \ldots, 50,000$.

Under our setup, we first perform the predictive regression in equation (A1) and we compute the Newey and West $p$-values, denoted by $p_{m}^{NW}$ (with automatically selected lags according to Newey and West (1994)). Next, based on the time-series in each simulation, we compute the Newey and West $p$-values and then the parametric bootstrap $p$-value, denoted by $p_{m}^{PB}$. The number of bootstrap runs for the parametric bootstrap is set to 20,000. In each simulation trial, we reestimate the parameters underlying $PC_{t}^{(8)}$ (while ensuring stationarity) and then bootstrap.

After 50,000 simulations, we compute the percentiles of $\left\{ p_{m}^{NW} \right\}_{m=1}^{50,000}$ and $\left\{ p_{m}^{PB} \right\}_{m=1}^{50,000}$.

If the negative MA component deteriorates the size of Newey and West statistical test, then one would expect that the 5th percentile of $p_{m}^{NW}$ be much less than 0.05. In contrast, if the parametric bootstrap procedure mitigates the small sample bias (e.g., Amihud, Hurvich, and Wang (2009, Appendix C)), then one would expect that the 5th percentile of $p_{m}^{PB}$ is closer to 0.05.

The evidence presented below shows that the size of the Newey and West estimator is distorted, even for the predictor $PC_{t}^{(9)}$, which is ARMA(0,0). Importantly, the size is remedied with the parametric bootstrap. For example, $PC_{t}^{(9)}$ displays the correct size, while $PC_{t}^{(8)}$ has a significantly improved size.
<table>
<thead>
<tr>
<th>Percentiles</th>
<th>PC_{t}^{(8)} , ARMA(1,1)</th>
<th>PC_{t}^{(9)} , ARMA(0,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st</td>
<td>5th</td>
</tr>
<tr>
<td>Newey and West</td>
<td>0.001</td>
<td>0.011</td>
</tr>
<tr>
<td>Parametric Bootstrap</td>
<td>0.009</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Overall, the simulation exercise reveals that the parametric bootstrap tends to provide conservative inference about the predictive slope coefficients in the setting of ARMA(1,1) predictor.
References


Bai, J., Ng, S., 2002. Determining the number of factors in approximate factor models. Econometrica 70, 191–221.


Sepp, A., 2008. VIX option pricing in a jump-diffusion model. Risk, April, 84–89.


Table 1

Size of uncertainty of uncertainty

All reported numbers are based on the prices of out-of-the-money (OTM) calls and puts on the VIX futures. The average days to expiration of the VIX options is 30 days. Reported first are the features of (i) uncertainty of uncertainty, \( uou_t \), as presented in equations (5)–(6), (ii) upside uncertainty of uncertainty, \( uou_t^+ \), as presented in equation (7). Next, we compute tail asymmetry as

\[
\text{asymmetry}_t = \frac{\mathbb{E}_t^* (z_{t+1}^3)}{\mathbb{E}_t^* (z_{t+1}^2)^{3/2}},
\]

where \( \mathbb{E}_t^* (z_{t+1}^3) \) is presented in equation (14). Finally, tail size is computed as

\[
\text{tail size}_t = \mathbb{E}_t^* (z_{t+1}^4) - (\mathbb{E}_t^* (z_{t+1}^2))^2,
\]

as displayed in equations (5) and (13). The sample period is 5/18/2006 to 03/17/2016 for a total of 119 monthly expiration cycles. \( R_{f,t+1} \) is the gross return of a riskfree bond with maturity that is matched to the number of days in the expiration cycles of VIX options. \( \rho_i \) is the autocorrelation at lag \( i \). We report the 5th, 50th, and the 95th percentiles. \( uou_t, uou_t^+, \) and \( vvix_t \) are annualized and expressed in percentage units.

<table>
<thead>
<tr>
<th></th>
<th>( uou_t )</th>
<th>( uou_t^+ )</th>
<th>asymmetry(_t)</th>
<th>.tail size(_t)</th>
<th>( vvix_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>94.1</td>
<td>85.7</td>
<td>2.47</td>
<td>0.054</td>
<td>89.3</td>
</tr>
<tr>
<td>Std.</td>
<td>21.8</td>
<td>20.7</td>
<td>0.67</td>
<td>0.048</td>
<td>14.9</td>
</tr>
<tr>
<td>5th</td>
<td>61.2</td>
<td>52.7</td>
<td>1.47</td>
<td>0.008</td>
<td>69.1</td>
</tr>
<tr>
<td>50th</td>
<td>93.3</td>
<td>85.4</td>
<td>2.40</td>
<td>0.043</td>
<td>86.6</td>
</tr>
<tr>
<td>95th</td>
<td>129.7</td>
<td>117.3</td>
<td>3.80</td>
<td>0.135</td>
<td>112.7</td>
</tr>
<tr>
<td>Min.</td>
<td>48.9</td>
<td>40.3</td>
<td>1.18</td>
<td>0.005</td>
<td>62.0</td>
</tr>
<tr>
<td>Max.</td>
<td>172.2</td>
<td>153.9</td>
<td>4.32</td>
<td>0.289</td>
<td>155.4</td>
</tr>
</tbody>
</table>

\[
\rho_1 = 0.59, \quad \rho_3 = 0.44, \quad \rho_6 = 0.24, \quad \rho_9 = 0.17, \quad \rho_{12} = 0.22
\]

\[
\rho_1 = 0.60, \quad \rho_3 = 0.48, \quad \rho_6 = 0.33, \quad \rho_9 = 0.25, \quad \rho_{12} = 0.25
\]
Table 2
Correlations of uncertainty of uncertainty with the macroeconomic uncertainty index, economic policy uncertainty index, the distress index, and the VIX

We report the correlations, $\phi_{\ell}$, between $uou_{t-1}$, $uou_{t}$, $uou_{t+1}$ (or $uou_{t-1}^+, uou_{t}^+, uou_{t+1}^+$) with (i) the macroeconomic uncertainty index of Jurado, Ludvigson, and Ng (2015, equation (1)), (ii) the economic policy uncertainty index of Baker, Bloom, and Davis (2015), (iii) the distress index constructed by the Federal Reserve Bank of St. Louis, and (iv) the VIX (daily averaged over the expiration cycles). The sample period is 5/18/2006 to 3/17/2016. The data on the macroeconomic uncertainty index is taken from the website of Sydney Ludvigson, while the economic policy uncertainty index is taken from www.policyuncertainty.com. We are careful to align the data when computing the correlations. The 95% lower and upper bootstrap confidence intervals are shown as Lower CI and Upper CI, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$uou_{\ell}$</th>
<th></th>
<th></th>
<th>$uou_{\ell}^+$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t-1$</td>
<td>$t$</td>
<td>$t+1$</td>
<td>$t-1$</td>
<td>$t$</td>
<td>$t+1$</td>
</tr>
<tr>
<td>Jurado, Ludvigson, and Ng (macroeconomic uncertainty index)</td>
<td>$\phi_{\ell}$</td>
<td>-0.27</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.35</td>
<td>-0.32</td>
</tr>
<tr>
<td>(macroeconomic uncertainty index)</td>
<td>Lower CI</td>
<td>-0.43</td>
<td>-0.41</td>
<td>-0.40</td>
<td>-0.48</td>
<td>-0.46</td>
</tr>
<tr>
<td></td>
<td>Upper CI</td>
<td>-0.10</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.20</td>
<td>-0.18</td>
</tr>
<tr>
<td>Baker, Bloom, and Davis (economic policy uncertainty index)</td>
<td>$\phi_{\ell}$</td>
<td>0.30</td>
<td>0.34</td>
<td>0.26</td>
<td>0.27</td>
<td>0.31</td>
</tr>
<tr>
<td>(economic policy uncertainty index)</td>
<td>Lower CI</td>
<td>0.16</td>
<td>0.21</td>
<td>0.11</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>Upper CI</td>
<td>0.43</td>
<td>0.46</td>
<td>0.40</td>
<td>0.41</td>
<td>0.44</td>
</tr>
<tr>
<td>Distress index</td>
<td>$\phi_{\ell}$</td>
<td>-0.20</td>
<td>-0.21</td>
<td>-0.29</td>
<td>-0.29</td>
<td>-0.30</td>
</tr>
<tr>
<td>(averaged over expiration cycles)</td>
<td>Lower CI</td>
<td>-0.36</td>
<td>-0.38</td>
<td>-0.43</td>
<td>-0.42</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>Upper CI</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-0.14</td>
<td>-0.15</td>
<td>-0.15</td>
</tr>
<tr>
<td>VIX (averaged over expiration cycles)</td>
<td>$\phi_{\ell}$</td>
<td>0.00</td>
<td>0.08</td>
<td>0.02</td>
<td>-0.08</td>
<td>-0.01</td>
</tr>
<tr>
<td>(averaged over expiration cycles)</td>
<td>Lower CI</td>
<td>-0.16</td>
<td>-0.08</td>
<td>-0.14</td>
<td>-0.22</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>Upper CI</td>
<td>0.16</td>
<td>0.24</td>
<td>0.18</td>
<td>0.07</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Table 3  
Uncertainty of uncertainty and principal components (the predictors) extracted from a large macroeconomic data set  

The dependent variable in the regression is uncertainty of uncertainty, denoted by $uou_t$ (annualized and expressed in percentage units). Reported results are based on the OLS regression:

$$uou_t = \beta_0 + \beta_2 PC^{(2)}_{t-1} + \beta_3 PC^{(3)}_{t-1} + \beta_8 PC^{(8)}_{t-1} + \beta_9 PC^{(9)}_{t-1} + \epsilon_t,$$

where $PC^{(j)}_{t-1}$ are the estimated principal components. We select the specification with $PC^{(2)}_{t-1}$, $PC^{(3)}_{t-1}$, $PC^{(8)}_{t-1}$, and $PC^{(9)}_{t-1}$, because this combination of principal components delivers the lowest BIC out of $2^{10} = 1024$ regressions (in contrast to a BIC of 6.193 with the first 10 principal components as predictors). To correct for autocorrelation and heteroscedasticity, we use the Newey and West (1987) estimator with automatically selected lag, denoted by $\ell^*$, as in Newey and West (1994), and the reported two-sided $p$-values are denoted by NW$[p]$. Shown next are the Hodrick (1992) two-sided $p$-values, denoted by H$[p]$. Finally, PB$[p]$ are the $p$-values from the parametric bootstrap. The adjusted $R^2$ is reported as $\bar{R}^2$. The intercept $\beta_0$ is included in the regression, but is not reported. We record the joint $p$-values for the null hypothesis that the slope coefficients are all equal to zero.

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW$[p]$</td>
<td>(0.001)</td>
<td>(0.017)</td>
<td>(0.048)</td>
<td>(0.000)</td>
<td>(6.0245)</td>
<td>6</td>
<td>0.000</td>
</tr>
<tr>
<td>H$[p]$</td>
<td>[0.002]</td>
<td>[0.009]</td>
<td>[0.037]</td>
<td>[0.000]</td>
<td></td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>PB$[p]$</td>
<td>{0.007}</td>
<td>{0.097}</td>
<td>{0.067}</td>
<td>{0.000}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW$[p]$</td>
<td>(0.012)</td>
<td>(0.080)</td>
<td>(0.000)</td>
<td></td>
<td>(6.0477)</td>
<td>7</td>
<td>0.000</td>
</tr>
<tr>
<td>H$[p]$</td>
<td>[0.016]</td>
<td>[0.065]</td>
<td>[0.000]</td>
<td></td>
<td></td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>PB$[p]$</td>
<td>{0.024}</td>
<td>{0.105}</td>
<td>{0.000}</td>
<td></td>
<td></td>
<td>0.007</td>
<td></td>
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<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW$[p]$</td>
<td>(0.018)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td>(6.0480)</td>
<td>3</td>
<td>0.000</td>
</tr>
<tr>
<td>H$[p]$</td>
<td>[0.020]</td>
<td>[0.000]</td>
<td></td>
<td></td>
<td></td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>PB$[p]$</td>
<td>{0.037}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.003</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW$[p]$</td>
<td>(0.087)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td>(6.0570)</td>
<td>7</td>
<td>0.000</td>
</tr>
<tr>
<td>H$[p]$</td>
<td>[0.075]</td>
<td>[0.000]</td>
<td></td>
<td></td>
<td></td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>PB$[p]$</td>
<td>{0.114}</td>
<td>{0.002}</td>
<td></td>
<td></td>
<td></td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>
Table 4
Forecasting uncertainty of uncertainty, when the predictors are lagged by two periods

The dependent variable in the predictive regression is uncertainty of uncertainty, denoted by $u_{ou_t}$ (annualized and expressed in percentage units). Reported results are based on the OLS regression:

$$u_{ou_t} = \beta_0 + \beta_2 PC_{t-2}^{(2)} + \beta_3 PC_{t-2}^{(3)} + \beta_8 PC_{t-2}^{(8)} + \beta_9 PC_{t-2}^{(9)} + \varepsilon_t,$$

where $PC_{t-2}^{(j)}$ are the estimated principal components that are lagged twice. Specifically, if we are forecasting $u_{ou_t}$ on 3/17/2016, then we use the principal components for the month of January 2016. To correct for autocorrelation and heteroscedasticity, we use the Newey and West (1987) estimator with automatically selected lag, denoted by $\ell^*$, as in Newey and West (1994), and the reported two-sided $p$-values are denoted by NW[$p$]. Shown next are the Hodrick (1992) two-sided $p$-values, denoted by H[$p$]. Finally, PB[$p$] are the $p$-values from the parametric bootstrap. The adjusted $R^2$ is reported as $\bar{R}^2$. The intercept $\beta_0$ is included in the regression, but is not reported. We record the joint $p$-values for the null hypothesis that the slope coefficients are all equal to zero.

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW[$p$]</td>
<td>(0.000)</td>
<td>(0.011)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>18.1%</td>
<td>3</td>
<td>0.000</td>
</tr>
<tr>
<td>H[$p$]</td>
<td>[0.004]</td>
<td>[0.010]</td>
<td>[0.001]</td>
<td>[0.002]</td>
<td>[0.004]</td>
<td>5</td>
<td>0.000</td>
</tr>
<tr>
<td>PB[$p$]</td>
<td>{0.011}</td>
<td>{0.131}</td>
<td>{0.012}</td>
<td>{0.009}</td>
<td>{0.015}</td>
<td>6</td>
<td>0.009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW[$p$]</td>
<td>(0.021)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td></td>
<td>12.2%</td>
<td>5</td>
<td>0.003</td>
</tr>
<tr>
<td>H[$p$]</td>
<td>[0.062]</td>
<td>[0.002]</td>
<td>[0.002]</td>
<td>[0.003]</td>
<td>[0.014]</td>
<td>7</td>
<td>0.015</td>
</tr>
<tr>
<td>PB[$p$]</td>
<td>{0.061}</td>
<td>{0.143}</td>
<td>{0.012}</td>
<td>{0.009}</td>
<td>{0.046}</td>
<td>9</td>
<td>0.009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW[$p$]</td>
<td>(0.066)</td>
<td>(0.006)</td>
<td></td>
<td></td>
<td>6.7%</td>
<td>4</td>
<td>0.009</td>
</tr>
<tr>
<td>H[$p$]</td>
<td>[0.082]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.013]</td>
<td>6</td>
<td>0.003</td>
</tr>
<tr>
<td>PB[$p$]</td>
<td>{0.120}</td>
<td>{0.131}</td>
<td>{0.012}</td>
<td>{0.009}</td>
<td>7</td>
<td>0.046</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_8$</th>
<th>$\beta_9$</th>
<th>$\bar{R}^2$</th>
<th>$\ell^*$</th>
<th>Joint $p$-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW[$p$]</td>
<td>(0.002)</td>
<td>(0.005)</td>
<td></td>
<td></td>
<td>10.2%</td>
<td>6</td>
<td>0.004</td>
</tr>
<tr>
<td>H[$p$]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.024]</td>
<td>8</td>
<td>0.000</td>
</tr>
<tr>
<td>PB[$p$]</td>
<td>{0.024}</td>
<td>{0.011}</td>
<td>{0.012}</td>
<td>{0.009}</td>
<td>10</td>
<td>0.031</td>
<td></td>
</tr>
</tbody>
</table>
Table 5

Relation between the macroeconomic primitives and principal components

The Macroeconomic primitive $v_t$ are variables included in the data set of McCraken and Ng (2015). The data is divided into eight groups: (1) output and income; (2) labor market; (3) housing; (4) consumption orders, and inventories; (5) money and credit; (6) interest and exchange rates; (7) prices; and (8) stock market. We perform the regressions: Macroeconomic primitive $a + bPC_{j,t} + e_{i,n}$, for $j = 2, 3, 8, 9,$ and $n = 1, \ldots, 123$. We report the group number (1 through 8), the identifier (ID) of the macroeconomic primitive, the adjusted $R^2$, the pairwise correlation $\phi$, and the name of the macroeconomic primitive. For compactness, we present the 15 variables with the highest $R^2$. The variable transformation is shown with $\Delta$ (second) difference of a series.

<table>
<thead>
<tr>
<th>Group</th>
<th>ID</th>
<th>$R^2$</th>
<th>$\phi$</th>
<th>Name (transformation)</th>
<th>Group</th>
<th>ID</th>
<th>$R^2$</th>
<th>$\phi$</th>
<th>Name (transformation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>122</td>
<td>0.71</td>
<td>-0.85</td>
<td>CPI, all items less medical care ($\Delta^2$ log)</td>
<td>6</td>
<td>101</td>
<td>0.37</td>
<td>0.61</td>
<td>Trade weighted U.S. dollar index, major currencies ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>113</td>
<td>0.71</td>
<td>-0.85</td>
<td>CPI, all items ($\Delta^2$ log)</td>
<td>6</td>
<td>102</td>
<td>0.31</td>
<td>0.56</td>
<td>Switzerland/US foreign exchange rate ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>120</td>
<td>0.70</td>
<td>-0.84</td>
<td>CPI, all items less food ($\Delta^2$ log)</td>
<td>6</td>
<td>104</td>
<td>0.22</td>
<td>0.48</td>
<td>UK/US foreign exchange rate ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>117</td>
<td>0.70</td>
<td>-0.84</td>
<td>CPI, commodities ($\Delta^2$ log)</td>
<td>6</td>
<td>103</td>
<td>0.20</td>
<td>0.45</td>
<td>Japan/US foreign exchange rate ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
<td>0.69</td>
<td>-0.83</td>
<td>Pers. cons. expend., nondurable goods ($\Delta^2$ log)</td>
<td>2</td>
<td>26</td>
<td>0.19</td>
<td>-0.45</td>
<td>Average duration of unemployment ($\Delta$)</td>
</tr>
<tr>
<td>7</td>
<td>115</td>
<td>0.69</td>
<td>-0.83</td>
<td>CPI, transportation ($\Delta^2$ log)</td>
<td>2</td>
<td>26</td>
<td>0.16</td>
<td>0.41</td>
<td>Average weekly hours, goods producing</td>
</tr>
<tr>
<td>2</td>
<td>123</td>
<td>0.67</td>
<td>-0.82</td>
<td>Pers. cons. expend., chain index ($\Delta^2$ log)</td>
<td>PC</td>
<td>4</td>
<td>65</td>
<td>0.14</td>
<td>-0.39 New orders for durable goods ($\Delta$ log)</td>
</tr>
<tr>
<td>2</td>
<td>108</td>
<td>0.60</td>
<td>-0.78</td>
<td>PPI, intermediate materials ($\Delta^2$ log)</td>
<td>8</td>
<td>6</td>
<td>105</td>
<td>0.14</td>
<td>0.38 Canada/US foreign exchange rate ($\Delta$ log)</td>
</tr>
<tr>
<td>2</td>
<td>121</td>
<td>0.59</td>
<td>-0.77</td>
<td>CPI, all items less shelter ($\Delta^2$ log)</td>
<td>2</td>
<td>48</td>
<td>0.14</td>
<td>0.38</td>
<td>Average weekly hours, manufacturing</td>
</tr>
<tr>
<td>2</td>
<td>107</td>
<td>0.58</td>
<td>-0.77</td>
<td>PPI, finished consumer goods ($\Delta^2$ log)</td>
<td>2</td>
<td>31</td>
<td>0.12</td>
<td>-0.36</td>
<td>Civilian unemployed for 27 weeks and over ($\Delta$ log)</td>
</tr>
<tr>
<td>2</td>
<td>106</td>
<td>0.58</td>
<td>-0.76</td>
<td>PPI, finished goods ($\Delta^2$ log)</td>
<td>4</td>
<td>66</td>
<td>0.12</td>
<td>-0.36</td>
<td>New orders for non-defence capital goods ($\Delta$ log)</td>
</tr>
<tr>
<td>2</td>
<td>109</td>
<td>0.57</td>
<td>-0.53</td>
<td>PPI, crude materials ($\Delta^2$ log)</td>
<td>2</td>
<td>29</td>
<td>0.11</td>
<td>-0.35</td>
<td>Civilians unemployed, 15 weeks and over ($\Delta$ log)</td>
</tr>
<tr>
<td></td>
<td>98</td>
<td>0.20</td>
<td>0.46</td>
<td>10 Year Treasury minus FED funds</td>
<td>7</td>
<td>112</td>
<td>0.10</td>
<td>-0.33</td>
<td>ISM manufacturing, price index</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>0.20</td>
<td>-0.45</td>
<td>Crude oil, spiced WTI and cushing ($\Delta^2$ log)</td>
<td>5</td>
<td>71</td>
<td>0.10</td>
<td>-0.33</td>
<td>M2 money stock ($\Delta^2$ log)</td>
</tr>
<tr>
<td>6</td>
<td>99</td>
<td>0.20</td>
<td>0.45</td>
<td>Moody’s Baa corporate minus Fed funds</td>
<td>5</td>
<td>76</td>
<td>0.10</td>
<td>-0.31</td>
<td>Commercial and industrial loans ($\Delta^2$ log)</td>
</tr>
<tr>
<td>7</td>
<td>113</td>
<td>0.55</td>
<td>0.74</td>
<td>CPI, all items ($\Delta^2$ log)</td>
<td>1</td>
<td>2</td>
<td>0.17</td>
<td>0.42</td>
<td>Real personal income ex. transfer receipts ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>122</td>
<td>0.55</td>
<td>0.74</td>
<td>CPI, all items less medical care ($\Delta^2$ log)</td>
<td>5</td>
<td>71</td>
<td>0.17</td>
<td>0.42</td>
<td>M2 money stock ($\Delta^2$ log)</td>
</tr>
<tr>
<td>7</td>
<td>120</td>
<td>0.54</td>
<td>0.74</td>
<td>CPI, all items less food ($\Delta^2$ log)</td>
<td>2</td>
<td>24</td>
<td>0.14</td>
<td>0.38</td>
<td>Civilian employment ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>117</td>
<td>0.54</td>
<td>0.74</td>
<td>CPI, commodities ($\Delta^2$ log)</td>
<td>2</td>
<td>36</td>
<td>0.13</td>
<td>0.38</td>
<td>All Employees, construction ($\Delta$ log)</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
<td>0.54</td>
<td>0.74</td>
<td>Pers. cons. expend., nondurable ($\Delta^2$ log)</td>
<td>2</td>
<td>46</td>
<td>0.13</td>
<td>0.38</td>
<td>Average weekly hours, goods producing</td>
</tr>
<tr>
<td>7</td>
<td>123</td>
<td>0.53</td>
<td>0.73</td>
<td>Pers. cons. expend., chain index ($\Delta^2$ log)</td>
<td>1</td>
<td>1</td>
<td>0.13</td>
<td>0.37</td>
<td>Real personal income ex. transfer receipts ($\Delta$ log)</td>
</tr>
<tr>
<td>PC</td>
<td>115</td>
<td>0.53</td>
<td>0.73</td>
<td>transportation ($\Delta^2$ log)</td>
<td>PC</td>
<td>5</td>
<td>74</td>
<td>0.13</td>
<td>0.37</td>
</tr>
<tr>
<td>9</td>
<td>121</td>
<td>0.45</td>
<td>0.68</td>
<td>CPI, all items less shelter ($\Delta^2$ log)</td>
<td>2</td>
<td>25</td>
<td>0.12</td>
<td>-0.35</td>
<td>Civilian unemployment rate ($\Delta$)</td>
</tr>
<tr>
<td>9</td>
<td>107</td>
<td>0.44</td>
<td>0.67</td>
<td>PPI, finished consumer goods ($\Delta^2$ log)</td>
<td>2</td>
<td>44</td>
<td>0.11</td>
<td>0.35</td>
<td>All employees, financial activities ($\Delta$ log)</td>
</tr>
<tr>
<td>9</td>
<td>106</td>
<td>0.44</td>
<td>0.67</td>
<td>PPI, finished goods ($\Delta^2$ log)</td>
<td>5</td>
<td>70</td>
<td>0.11</td>
<td>0.35</td>
<td>M1 money stock ($\Delta^2$ log)</td>
</tr>
<tr>
<td>9</td>
<td>108</td>
<td>0.41</td>
<td>0.65</td>
<td>PPI, intermediate materials ($\Delta^2$ log)</td>
<td>2</td>
<td>27</td>
<td>0.11</td>
<td>-0.34</td>
<td>Civilians unemployed, less than 5 weeks ($\Delta$ log)</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>0.39</td>
<td>-0.63</td>
<td>Housing starts, total new privately owned (log)</td>
<td>7</td>
<td>112</td>
<td>0.10</td>
<td>-0.33</td>
<td>ISM manufacturing, price index</td>
</tr>
<tr>
<td>3</td>
<td>53</td>
<td>0.39</td>
<td>-0.63</td>
<td>Housing starts-south (log)</td>
<td>2</td>
<td>48</td>
<td>0.09</td>
<td>0.32</td>
<td>Average weekly hours, manufacturing</td>
</tr>
<tr>
<td>3</td>
<td>52</td>
<td>0.38</td>
<td>-0.62</td>
<td>Housing starts-midwest (log)</td>
<td>5</td>
<td>131</td>
<td>0.09</td>
<td>0.31</td>
<td>MZM money stock ($\Delta^2$ log)</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
<td>0.35</td>
<td>-0.60</td>
<td>New private housing permits-midwest (log)</td>
<td>2</td>
<td>34</td>
<td>0.09</td>
<td>0.31</td>
<td>All employees, goods producing industries ($\Delta$ log)</td>
</tr>
</tbody>
</table>
Table 6

Forecasting uncertainty of uncertainty with individual macroeconomic primitives

The dependent variable in the predictive regression is uncertainty of uncertainty, denoted by \( uou_t \) (annualized and expressed in percentage units). Reported results are based on the OLS regression:

\[
    uou_t = \theta_0 + \theta_1 \log(x_{t-1}^1) + \theta_2 \Delta \log(x_{t-1}^2) + \\
    \theta_3 \Delta \log(x_{t-1}^3) + \theta_4 \Delta \log(x_{t-1}^4) + \varepsilon_t,
\]

where the predictors are

- \( x_{t-1}^1 \): Housing starts-midwest (ID : 52, belongs to PC\(^{(3)}\)),
- \( x_{t-1}^2 \): Switzerland/US foreign exchange rate (ID : 102, belongs to PC\(^{(8)}\)),
- \( x_{t-1}^3 \): Real personal income excluding transfer receipts (ID : 2, belongs to PC\(^{(9)}\)),
- \( x_{t-1}^4 \): All employees, financial activities (ID : 44, belongs to PC\(^{(9)}\)).

We select the above specification via a two-step algorithm. First, we perform univariate predictive regressions with the 15 variables that have the highest \( R^2 \), each in PC\(^{(2)}\), PC\(^{(3)}\), PC\(^{(8)}\), and PC\(^{(9)}\), as in Table 5 (so a total of 60 regressions). This step isolates 18 macroeconomic primitives that have \( p \)-values lower than 0.1, according to Newey and West (1987) (with automatically selected lag). Second, we select the variable combination that yields the lowest BIC out of 2\(^{18} \cdot \) 2\(^{144} \)= 2\(^{578} \) regressions. To correct for autocorrelation and heteroscedasticity, we use the Newey and West (1987) estimator with automatically selected lag, denoted by \( \ell^* \), as in Newey and West (1994), and the reported two-sided \( p \)-values are denoted by NW\([p]\). Shown next are the Hodrick (1992) two-sided \( p \)-values, denoted by H\([p]\). Finally, PB\([p]\) are the \( p \)-values from the parametric bootstrap. The parametric bootstrap takes into account that \( \log(x_{t}^1), \Delta \log(x_{t}^2), \Delta \log(x_{t}^3), \) and \( \Delta \log(x_{t}^4) \), respectively, follow ARMA(1,1), ARMA(0,0), ARMA(0,0), and AR(2) process. The intercept \( \theta_0 \) is included in the regression, but is not reported. We record the joint \( p \)-values for the null hypothesis that \( \theta_1 = \theta_2 = \theta_3 = \theta_4 = 0 \), the standard deviation of the predictors (Std.), and the Std. multiplied by the \( \theta \) estimate (i.e., the sensitivity of \( uou_t \) to a one-standard deviation change in each predictor).

<table>
<thead>
<tr>
<th>ID</th>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
<th>( \theta_3 )</th>
<th>( \theta_4 )</th>
<th>( R^2 )</th>
<th>( \ell^* )</th>
<th>Joint ( p )-val.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>NW([p])</td>
<td>H([p])</td>
<td>PB([p])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>-23</td>
<td>(0.000)</td>
<td>(0.004)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>0.000</td>
<td>5.9898</td>
</tr>
<tr>
<td>102</td>
<td>184</td>
<td>(0.001)</td>
<td>(0.020)</td>
<td>(0.024)</td>
<td>(0.000)</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>495</td>
<td>(0.000)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>4560</td>
<td>(0.000)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Std., predictor 0.357 0.026 0.008 0.002
Std*\( \theta \) -8.19 4.79 4.13 9.07
### Table Appendix-I

**Number of strikes, trading volume, and open interest for nearest-maturity options on the VIX futures**

Reported are the number of out-of-the-money (OTM) strikes, the trading volume, and open interest in nearest-maturity options on the VIX futures, recorded at the beginning of the expiration cycles. The sample period is 5/18/2006 to 3/17/2016, with 119 monthly expiration cycles. The source of the options data is OptionMetrics. The sample contains a total of 1425 OTM calls and 645 OTM puts. Trading volume and open interest are stated as number of contracts.

<table>
<thead>
<tr>
<th></th>
<th>Number of OTM strikes</th>
<th>Trading volume</th>
<th>Open interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calls</td>
<td>Puts</td>
<td>Calls</td>
</tr>
<tr>
<td>Mean</td>
<td>12.0</td>
<td>5.4</td>
<td>124,857</td>
</tr>
<tr>
<td>Std.</td>
<td>4.1</td>
<td>1.0</td>
<td>120,576</td>
</tr>
</tbody>
</table>
Table Appendix-II

Statistical analysis and model selection results for the principal components

To select $PC_t^{(2)}$, $PC_t^{(3)}$, $PC_t^{(8)}$, and $PC_t^{(9)}$ as the predictors, we follow two steps. First, we determine the number of principal components using the procedure advocated by Bai and Ng (2002), applied to the 123 macroeconomic time series from McCraken and Ng (2015), over the sample period of January 1978 to February 2016. We appropriately transform and standardize the data to have zero mean and unit variance prior to implementing the panel information criterion. This procedure yields 10 principal components, denoted $PC_t^{(j)}$ for $j = 1, \ldots, 10$, and explains 56.7% of the variance. Second, we minimize the BIC across predictor combinations over the 1,024 possible predictive regressions in equation (15). This procedure selects four predictors: $PC_t^{(2)}$, $PC_t^{(3)}$, $PC_t^{(8)}$, and $PC_t^{(9)}$. Following Greene (2011, Chapter 5, equation (5-44)), we compute the BIC as $\log(\varepsilon'\varepsilon/T) + k_0 \log(T)/T$, where $T$ is the number of observations and $k_0$ is the number of parameters. We report the autocorrelations and the $p$-values from the unit-root tests for each of the four predictors. Also displayed are the parameter estimates of the best model, selected according to the BIC criterion, among ARMA($p$, $q$) models with $p = 3$ and $q = 3$.

<table>
<thead>
<tr>
<th></th>
<th>$PC_t^{(2)}$</th>
<th>$PC_t^{(3)}$</th>
<th>$PC_t^{(8)}$</th>
<th>$PC_t^{(9)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>0.10</td>
<td>0.33</td>
<td>0.18</td>
<td>0.09</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>0.05</td>
<td>0.28</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>$\rho_6$</td>
<td>0.10</td>
<td>0.26</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>$\rho_9$</td>
<td>0.14</td>
<td>0.28</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>-0.01</td>
<td>0.14</td>
<td>0.08</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Panel A: Autocorrelations

Panel B: Unit-root tests, $p$-values

<table>
<thead>
<tr>
<th></th>
<th>Augmented Dickey and Fuller</th>
<th>Phillips and Perron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>0.080</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Panel C: Model selection according to the Bayesian information criterion

<table>
<thead>
<tr>
<th>Model</th>
<th>Coeff.</th>
<th>ARMA(0,0)</th>
<th>ARMA(1,1)</th>
<th>ARMA(1,1)</th>
<th>ARMA(0,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>(p-val.)</td>
<td>0.99</td>
<td>0.97</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>MA(1)</td>
<td>(p-val.)</td>
<td>-0.83</td>
<td>-0.88</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>
Figure 1. **Variations in uncertainty of uncertainty**

We plot the uncertainty of uncertainty, computed as

$$ uou_t^2 = \int_{K>F_t} \left( \frac{2R_{t,t+1}}{F_t^2} \right) C_t[K] dK + \int_{K<F_t} \left( \frac{2R_{t,t+1}}{F_t^2} \right) P_t[K] dK, $$

and the integrated variance of VIX futures returns, computed as

$$ vvix_t^2 = \int_{K>F_t} \left( \frac{2R_{t,t+1}}{K^2} \right) C_t[K] dK + \int_{K<F_t} \left( \frac{2R_{t,t+1}}{K^2} \right) P_t[K] dK. $$

The sample period is 5/18/2006 to 3/17/2016 (119 monthly expiration cycles). The grey shaded regions represent recessions (as classified by the NBER). The source of the options on the VIX futures is OptionMetrics. Both $uou_t$ and $vvix_t$ are annualized and expressed in percentage units.
We plot the time variation in the four principal components: \( \text{PC}_t^{(2)} \), \( \text{PC}_t^{(3)} \), \( \text{PC}_t^{(8)} \), and \( \text{PC}_t^{(9)} \). The sample period is April 2006 to February 2016. The grey shaded regions represent recessions (as classified by the NBER). To select \( \text{PC}_t^{(2)} \), \( \text{PC}_t^{(3)} \), \( \text{PC}_t^{(8)} \), and \( \text{PC}_t^{(9)} \) as the predictors, we follow two steps. First, we determine the number of principal components using the procedure advocated by Bai and Ng (2002), applied to the 123 macroeconomic time series from McCraken and Ng (2015). We appropriately transform and standardize the data to have zero mean and unit variance prior to implementing the panel information criterion. This procedure yields 10 principal components, denoted \( \text{PC}_t^{(j)} \) for \( j = 1, \ldots, 10 \). Second, we minimize the BIC across predictor combinations over the 1,024 possible predictive regressions in equation (15).
Figure 3. **Goodness-of-fit $R^2$ obtained by regressing macroeconomic primitives on the principal components**

Displayed is the group number (1 through 8, x-axis) and the adjusted $R^2$ (y-axis). Computing the $R^2$ entails running the following regression: $\text{Macroeconomic primitive}_{t,n} = a + b \text{PC}_{t}^{(j)} + \epsilon_{t,n}$, for $j = 2, 3, 8, 9$, and $n = 1, \ldots, 123$. The Macroeconomic primitive $\text{Macroeconomic primitive}_{t,n}$ are the variables included in the data set of McCraken and Ng (2015). The data is divided into eight groups: (1) output and income; (2) labor market; (3) housing; (4) consumption orders, and inventories; (5) money and credit; (6) interest and exchange rates; (7) prices; and (8) stock market.
Figure 4. The pairwise correlation between the macroeconomic primitives and the principal components

Displayed is the group number (1 through 8, x-axis) and the pairwise correlation (y-axis). The reported correlation is between the Macroeconomic primitive $t_n$ and $PC_t^{(j)}$, for $j = 2, 3, 8, 9$, and $n = 1, \ldots, 123$. The Macroeconomic primitive $t_n$ are the variables included in the data set of McCraken and Ng (2015). The data is divided into eight groups: (1) output and income; (2) labor market; (3) housing; (4) consumption orders, and inventories; (5) money and credit; (6) interest and exchange rates; (7) prices; and (8) stock market.