Asset Pricing Implications of Short-sale Constraints in Imperfectly Competitive Markets *

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JEL Classification Codes: G11, G12, G14, D82.

Keywords: short-sale constraints; bid-ask spread; market liquidity; imperfect competition.

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Abstract

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1. Introduction

Implicit and explicit short-sale constraints are prevalent in many financial markets, and competition in most of these markets is far from perfect (e.g., Christie and Schultz (1994) and Biais, Bisière and Spatt (2010)). However, to the best of our knowledge, extant theories regarding how short-sale constraints affect asset prices and market liquidity exclusively focus on perfectly competitive markets.[1] In addition, they cannot explain a robust empirical finding that impositions of regulatory short-sale bans cause significant increases in bid-ask spreads in many financial markets.[2] Regulatory short-sale bans are more likely imposed when market conditions have deteriorated significantly and a large number of investors can only trade with a small number of designated market-makers who have significant market power (e.g., Anand and Venkataraman (2016)).[3] This motivates us to study the impact of short-sale constraints in an imperfectly competitive market in which investors trade through a small number of designated market-makers with market power. We find that short-sale constraints have qualitatively different impacts in the presence of market power. In particular, our analysis predicts that short-sale constraints decrease bid prices and increase bid-ask spreads. In addition, if market makers are risk-averse, then short-sale constraints also increase ask prices. Furthermore, our model suggests that the impact of short-sale constraints tends to be greater in markets with more transparency.

More specifically, we consider an equilibrium model with three types of risk-averse investors: hedgers, non-hedgers, and a designated market-maker. Investors can trade one risk-free asset and one risky security. Hedgers have trading demand motivated by hedging. In addition, hedgers may observe a private signal about the risky security's future payoff, and thus may also have information-motivated trading demand. Both hedgers and non-hedgers are subject to short-sale constraints and trade through the designated market-maker. As in Goldstein, Li, and Yang (2014), because investors have different motives to trade, their reservation prices may differ,[4] which causes

[1] See, for example, Harrison and Kreps (1978), Diamond and Verrecchia (1987), Bai, Chang, and Wang (2006), Yuan (2006), and Wang (2016).

[2] See, for example, Beber and Pagano (2013), Boehmer, Jones and Zhang (2013), and Ang, Shtauber, and Tetlock (2013).

[3] This market power is necessary for compensating them for the increased risk during this period.

[4] The reservation price is the critical price, such that an investor buys (sells) the security if and only if the ask

trading in equilibrium.

Because short-sale constraints restrict sales, one might expect that bid prices increase in equilibrium, as predicted by most of the extant theories (e.g., Harrison and Kreps (1978), Yuan (2006), Wang (2016)). However, this is exactly the opposite of what we find. One key difference of our model from the extant literature is that competition among short-sellers' counterparty (i.e., the market-maker) is imperfect in our model. The intuition for our opposite result can be illustrated with a simple example. Suppose that, without short-sale constraints, the short-seller short-sells ten shares (at the bid) in equilibrium, but with short-sale constraints, the short-seller can only short-sell five shares. Because the optimal number of shares that the short-seller chooses to short decreases as the bid price decreases, a market-maker with market power can lower the bid price to the level at which the constraints just start to bind (i.e., at this lower bid price, the short-seller shorts five shares even when unconstrained). By doing this, the market-maker pays a lower price for the shares without any adverse impact on the number of shares she can buy (i.e., still five shares). Therefore, because of the market power of the market-maker, the equilibrium bid price is lower with short-sale constraints.[5] More generally, when some investors are restricted from selling more, if buyers do not have market power, they will then compete for the reduced supply, and thus drive up the equilibrium trading price, as found in the extant literature that considers competitive markets. On the other hand, if buyers have market power, then the equilibrium price goes down, as we show in this paper. This is because a lower price is better for buyers, and if it is set at the level at which short-sale constraints just start to bind, it does not affect the number of shares buyers can buy. Our paper is the first to demonstrate how short-sale constraints affect the price at which short-sales occur (i.e., the bid) critically depends on whether buyers have market power.

Because the market-maker buys less from short-sellers when short-sale constraints bind, she also sells less at the ask by charging a higher ask price to achieve the optimal inventory risk exposure. This results in a higher ask price and a smaller ask depth. The simplest example to explain the

(bid) is lower (higher) than this critical price.

[5]To pinpoint market-makers' market power as the cause of the opposite result, we show in Theorems 5 and 6 in Appendix B that keeping everything else the same as in our model, if market-makers did not have market power, then short-sale constraints would indeed increase equilibrium bid prices.

intuition is when the market-maker is infinitely risk-averse. In this case, the market-maker does not carry any inventory (and makes profit only from the spread). Therefore, when her purchase at the bid is reduced by short-sale constraints imposed on other investors, she reduces her sale by the same amount by charging a higher ask price to avoid any net inventory position.

On the other hand, if the market-maker were risk-neutral, then the change in the inventory risk due to the reduction of purchases at the bid caused by short-sale constraints would be irrelevant for her, and thus short-sale constraints would not affect the ask price or the ask depth. This demonstrates how short-sale constraints affect the ask price, and the ask depth critically depends on a market-maker's risk-aversion. However, the same intuition as previously stated would still apply for the determination of the bid price and the bid depth, and thus short-sale constraints would still lower the bid and increase the spread. Therefore, while the market-maker's market power is the key driving force behind the result that short-sale constraints decrease the bid price, the market-maker's aversion to inventory risk is the channel through which short-sale constraints increase the ask price and decrease the ask depth.[6]

We show that, even in the presence of information asymmetry, our main qualitative results still hold. More public disclosure about asset payoff reduces overall uncertainty, which can increase investors' trading demand. As a result, short-sale constraints bind more often and thus have a greater effect. Thus, our model predicts that, ceteris paribus, the adverse impact of short-sale constraints on prices and market liquidity is greater in more transparent markets.

Because the imposition of short-sale constraints may change the benefit of private information, we further study whether endogenizing information acquisition invalidates our main results. To this end, we assume that the cost for the private signal about the risky security payoff is an increasing and convex function of the signal's precision. We find that, for a large set of parameter values, our main results, such as the increase in the expected spread, remain valid. We also demonstrate that,

[6] Although our model focuses on short-sale constraints, our main results also apply to any constraints that restrict the amount of sales or purchases by non-market-makers through the same mechanism. For example, our model indicates that limits on long positions drive up ask prices, drive down bid prices, reduce bid and ask depths, and increase bid-ask spread volatility. This is because with reduced demand at the ask price due to the limits, the market-maker increases the ask price, decreases the ask depth, and if she is risk-averse, she also buys less to control the inventory risk by lowering the bid price.

even when short-sale constraints prevent some negative news from being revealed, our main results still hold. In addition, we show that an extension to a dynamic setting does not change our main results either. This is because the key driving force behind the main results is the market power, which can still exist even in a dynamic setting.

To the best of our knowledge, Diamond and Verrecchia (1987) (hereafter DV) is the only theoretical paper in the existing literature that examines the effect of short-sale constraints on bid-ask spreads. Because the uninformed are unlikely to short even without a short-sale ban (e.g., Boehmer, Jones and Zhang (2008)), DV predict that, if short-sales are banned, then bid-ask spreads will narrow, as Boehmer, Jones and Zhang (2013) point out. This is because the ban prevents the informed from shorting, and thus other traders will face less adverse selection after the ban. On the other hand, if there is no information asymmetry, then DV predict that short-sale constraints have no impact on the bid or the ask or the spread.[7] In contrast, the extant empirical literature finds that bid-ask spreads significantly increase as a result of the 2008 short-sale bans (e.g., Beber and Pagano (2013), Boehmer, Jones and Zhang (2013), Ang, Shtauber, and Tetlock (2013)), which is exactly what our model predicts. As most of the rational expectations models in market microstructure literature (e.g., Glosten and Milgrom (1985), Admati and Pfleiderer (1988)), DV consider a perfect competition market with risk-neutral market-makers. However, the presence of a market-maker's market power is an important characteristic in the markets studied by the above empirical work around the 2008 short-sale bans. One of the reasons for this market power is that other liquidity providers in normal times tend to exit markets during bad times, and only a small number of market-makers remain active (e.g., Anand and Venkataraman (2016)). The difference in the prediction of DV and that of ours highlights the importance of a market-maker's market power in affecting the impact of short-sale bans.

The remainder of the paper proceeds as follows. We first discuss the applicability of models in imperfectly competitive markets and provide an additional literature review in the next section. In Section 3, we present the model. In Section 4, we derive the closed-form equilibrium results. In

[7] If both the uninformed and the informed short-sell before the ban, DV predict that, immediately after the imposition of the ban, there is no change in the bid or the ask, and thus the spread also remains the same. Over time, DV predict that the spread narrows more slowly, and thus becomes greater relative to that without the ban.

Section 5, we examine the effect of short-sale constraints on bid/ask prices, bid-ask spreads, and liquidity-risk. In Section 6, we show the robustness of the main results to various extensions, such as information asymmetry, endogenous information acquisition, reduced information revelation, and a dynamic setting. We conclude in Section 7. All proofs are in Appendix A. In Appendix B, we provide related results for the competitive case to identify the source of the difference of our results from those in the literature.

2. Applicable markets and additional related literature

Even relatively more liquid markets, such as NYSE, NASDAQ and Paris Bourse, employ designated market-makers to facilitate trading, especially during financial market stress.[8] These market-makers are required to maintain two-sided markets during exchange hours and are obligated to buy and sell at their displayed bids and offers. Designated market-makers are core liquidity providers in many of these markets, even under normal market conditions. For example, in 2015, designated market-makers accounted for about approximately 12% of liquidity adding volume in NYSE-listed securities, on average. Anand and Venkataraman (2016) find that endogenous liquidity providers scale back their participation in unison when market conditions are unfavorable. Around the imposition of the short-sale bans during the financial crisis in 2008, designated market-makers tend to play an even more important role in making the market because many endogenous liquidity providers become liquidity demanders at that time. Accordingly, to capture this feature, we focus on the trades that investors made with the designated market-makers to study the impact of short-sale constraints, although there are limit-order-book driven transactions in these markets.

Competition among market-makers is imperfect in many financial markets. For example, Christie and Schultz (1994) suggest that Nasdaq dealers may implicitly collude to maintain wide spreads. Biais, Bisière and Spatt (2010) analyze trades and order placement on Nasdaq and a competing electronic order book, Island. They conclude that competition among market-makers in these markets is still imperfect even after the introduction of electronic markets. In addition,

[8]See, for example, "Designated Market Makers," 2016, www.nyse.com, and Venkataraman and Waisb (2007).

the opaqueness and illiquidity of many dealers' markets make these markets even less competitive (e.g., Ang, Shtauber, and Tetlock (2013)).

Because there tend to be less liquidity and less trading volume in imperfectly competitive markets, implicit and explicit short-sale constraints are more prevalent in these markets. For example, short selling of small stocks is difficult and rare, possibly due to low ownership by market-makers and institutions (the main lenders of shares), which leads to high short-sale costs. Even though we model short-sale constraints in the form of an explicit limit on short positions instead of in the form of short-sale costs, the qualitative results from these two alternative approaches are the same if the short-sale costs are sufficiently high to reduce short sales, on average.[9] In addition, explicit short-sale constraints are also often imposed by market-making firms in many imperfectly competitive markets. For example, Ang, Shtauber, and Tetlock (2013) collect short-selling data for a sample of 50 OTC stocks and 50 similarly-sized (small) listed stocks in June 2012. They find that short-sales are prohibited for a large number of the listed stocks and even more for the OTC stocks. For instance, they state that "a retail customer of Fidelity could buy all 100 of these stocks, but the broker would allow short selling in only one of the OTC stocks and eight of the listed stocks."

A vast literature exists on the impact of short-sale constraints on asset prices in competitive markets. Most of these models, except Hong and Stein (2003) and Bai et al. (2006), find that short-sale constraints drive up trading prices (e.g., Scheinkman and Xiong (2003), and Wang (2016)). Hong and Stein (2003) and Bai et al. (2006) show that short-sale constraints can cause trading prices to go down, as in our model. However, the driving force in Hong and Stein (2003) and Bai et al. (2006) is the assumption that short-sale constraints prevent some investors from revealing negative information. For example, when the negative information initially prevented from being revealed is disclosed later, prices decrease, as shown in Hong and Stein (2003). In contrast, the driving force behind our result that short-sale constraints can lower trading prices is buyers' market power, and therefore our result holds even when there is no information asymmetry. Consequently, different from these two papers, our paper predicts that bid price decreases by a greater amount in more transparent markets. In addition, neither Hong and Stein (2003) nor Bai et al. (2006)

[9] As an extreme example, if short-sale costs are infinity, then it is equivalent to imposing short-sale bans.

examines the impact of short-sale constraints on bid-ask spreads. Liu and Wang (2016) study market-making in the presence of asymmetric information and inventory risk, and demonstrate that bid-ask spreads may decrease with information asymmetry. Short-sale constraints are absent in Liu and Wang (2016), and thus they are silent on the impact of short-sale constraints on market prices and liquidity. Nezafat, Schroder, and Wang (2014) consider a partial equilibrium model with endogenous information acquisition and short-sale constraints. In contrast to our model, they do not study the impact of short-sale constraints on *equilibrium* asset prices or bid-ask spreads, and there are no strategic traders in their model.

3. The model

We consider a one period setting with dates 0 and 1.[10] There are a continuum of identical hedgers with mass N_h, a continuum of identical non-hedgers with mass N_n, and $N_m = 1$ designated market-maker. They can trade one risk-free asset and one risky security on date 0 to maximize their expected constant absolute risk aversion (CARA) utility from the terminal wealth on date 1. No investor is endowed with any amount of the risk-free asset. The risk-free asset serves as the numeraire, and thus the risk-free interest rate is normalized to 0. For type $i \in \{h, n, m\}$ investors, the total risky security endowment is $N_i\bar{\theta}$ shares.[11] The aggregate supply of the risky security is $N \times \bar{\theta} > 0$ shares where $N = N_h + N_n + N_m$ and the date 1 payoff of each share is $\tilde{V}$, where $\tilde{V} \sim \mathbf{N}(\bar{V}, \sigma_V^2)$, $\bar{V}$ is a constant, $\sigma_V > 0$, and $\mathbf{N}(\cdot)$ denotes the normal distribution.

Hedgers are subject to a liquidity shock that is modeled as a random endowment of $\hat{X}_h \sim \mathbf{N}(0, \sigma_X^2)$ units of a non-tradable risky asset on date 0, with $\hat{X}_h$ realized on date 0.[12] The non-tradable asset has a per-unit payoff of $\tilde{L} \sim \mathbf{N}(0, \sigma_L^2)$ that has a covariance of σ_{VL} with the risky security's payoff $\tilde{V}$. The payoff of the non-tradable asset is realized and becomes public on date 1. The correlation between the non-tradable asset and the risky security results in a liquidity demand

[10] We show in Section 6.D that our main results still hold in a two-period dynamic setting.

[11] Given the CARA preferences, having different cash endowment would not change any of the results. Throughout this paper, "bar" variables are constants, "tilde" random variables are realized on date 1 and "hat" random variables are realized on date 0.

[12] The random endowment can represent any shock in the demand for the security, such as a liquidity shock or a change in the needs for rebalancing an existing portfolio or a change in a highly illiquid asset.

for the risky security to hedge the non-tradable asset payoff. The non-hedgers do not have any liquidity shocks, i.e., $\hat{X}_n = 0$.

All trades go through the designated market-maker.[13] As required by regulators, the designated market-maker must provide quotes on both sides of the market. Accordingly, we assume that the market-maker posts her price schedules first. Then hedgers and non-hedgers decide how much to sell to the designated market-maker at the bid B or buy from her at the ask A or do not trade at all. When deciding on what price schedules to post, the market-maker takes into account the best response functions (i.e., the demand schedules) of other investors given the to-be-posted price schedules.[14] In equilibrium, the risk-free asset market also clears.

We assume that both h and n investors are subject to short-sale constraints, i.e., the after-trade position $\theta_i + \bar{\theta} \geq -\kappa_i, \;\; i = h, n$, where $\kappa_i \geq 0$ can be different for the hedgers and the non-hedgers.[15] A smaller κ_i means a more stringent short-sale constraint: If $\kappa_i = 0$, then type i investors are prohibited from short selling; and if $\kappa_i = \infty$, then it is equivalent to the absence of short-sale constraints. Heterogeneous short-sale constraint stringencies for hedgers and non-hedgers capture the essence of possibly different short-sale costs across them and allow us to examine the impact of a short-sale ban when some investors cannot short-sell even without the ban (e.g., Kolasinki, Reed and Ringgenberg (2013)). In most markets, a designated market-maker is exempted from short-sale constraints by regulators to facilitate her liquidity provision. Accordingly, we assume that the market-maker is not subject to short-sale constraints.[16]

Because there is a continuum of hedgers and non-hedgers, we assume that they are price takers.[17]

[13] As demonstrated by Anand and Venkataraman (2016), many liquidity providers exit markets in bad times, and a large number of investors can only trade with a small number of designated market-makers. We assume zero market-making cost because a positive cost complicates analysis and does not change our main results as will become clear later.

[14] This is equivalent to a setting in which other investors submit demand schedules to the market-maker, similar to Kyle (1989), Glosten (1989), and Biais, Martimort, and Rochet (2000).

[15] An alternative way of modeling short-sale constraints is to impose short-sale costs. This alternative approach would yield the same qualitative results, because as the costs increase, the amount and frequency of short-sales decrease, which is qualitatively the same as the effect of decreasing the stringency parameter κ_i in our model.

[16] If the designated market-maker was also subject to short-sale constraints, then the qualitative results would stay the same. This is because the short-sale constraints for the market-maker restrict her sale at the ask. When the constraints bind for her, she cannot sell more at the ask price and therefore the ask price becomes higher, while the impact of the short-sale constraints on other investors remains qualitatively the same at the bid.

[17] Even if they had market power, the qualitative results would be the same, because short-sale constraints would still restrict their sales even when they have market power and the market power of the market-maker would still

After observing liquidity shock $\hat{X}_h$, each hedger chooses a demand schedule $\Theta_h(\hat{X}_h,\ \cdot)$. Because from equilibrium prices, non-hedgers can infer out the liquidity shock realized $\hat{X}_h$, there is no information asymmetry in equilibrium. Thus, each nonhedger chooses a demand schedule $\Theta_n(\hat{X}_h,\ \cdot)$ that can also directly depend on $\hat{X}_h$. The schedules Θ_h and Θ_n are traders' strategies. Given bid price B and ask price A, the quantities demanded by hedgers and non-hedgers can be written as $\theta_h = \Theta_h(\hat{X}_h, A, B)$ and $\theta_n = \Theta_n(\hat{X}_h, A, B)$.

Given A and B, for $i \in \{h, n\}$, a type i investor's problem is to choose θ_i to solve

$$\max E[-e^{-\delta \tilde{W}_i}], \tag{1}$$

subject to the budget constraint

$$\tilde{W}_i = \theta_i^- B - \theta_i^+ A + (\bar{\theta} + \theta_i)\tilde{V} + \hat{X}_i \tilde{L}, \tag{2}$$

and the short-sale constraint

$$\theta_i + \bar{\theta} \geq -\kappa_i, \tag{3}$$

where $\delta > 0$ is the absolute risk-aversion parameter, $\hat{X}_n = 0$, $x^+ := \max(0, x)$, and $x^- := \max(0, -x)$.[18]

Since h and n investors buy from the designated market-maker at ask and sell to her at bid, we can view these trades as occurring in two separate markets: the "ask" market and the "bid" market. In the ask market, the market-maker is the supplier, and other investors are demanders; and the opposite is true in the bid market. The monopolistic market-maker chooses bid and ask prices, taking into account other investors' demand curves in the ask market and supply curves in the bid market.

Given liquidity shock $\hat{X}_h$, let the realized demand schedules of hedgers and non-hedgers be denoted as $\Theta_h(A, B)$ and $\Theta_n(A, B)$ respectively, where A is the ask price and B is the bid price.

imply that the bid price goes down.

[18] We have solved the more general case in which investors have different liquidity shocks, different information, different initial endowment, and different risk-aversions, the results of which are available from the authors. Our main results on the impact of short-sale constraints are the same because the same intuition still applies.

By market-clearing conditions, the equilibrium ask depth α must be equal to the total amount bought by other investors, and the equilibrium bid depth β must be equal to the total amount sold by other investors, i.e., [19]

$$\alpha = \sum_{i=h,\ n} N_i \Theta_i(A,B)^+, \ \beta = \sum_{i=h,\ n} N_i \Theta_i(A,B)^-. \tag{4}$$

The risk-free asset market will be automatically cleared by the Walras' law. Note that, if an investor decides to buy (sell), then only the ask (bid) price affects how much he buys (sells), i.e., $\Theta_i(A,B)^+$ only depends on A and $\Theta_i(A,B)^-$ only depends on B. Therefore, the bid depth β only depends on B, henceforth referred as $\beta(B)$, and the ask depth α only depends on A, henceforth referred as $\alpha(A)$.

We denote the market-maker's pricing strategies as $\mathbb{A}(\ \cdot)$ and $\mathbb{B}(\ \cdot)$. For any realized demand schedules $\Theta_h(A,B)$ and $\Theta_n(A,B)$, the designated market-maker's problem is to choose an ask price level $A := \mathbb{A}(\Theta_h, \Theta_n)$ and a bid price level $B := \mathbb{B}(\Theta_h, \Theta_n)$ to solve[20]

$$\max E\left[-e^{-\delta \tilde{W}_m}\right], \tag{5}$$

subject to

$$\tilde{W}_m \ \ = \ \ \alpha(A)A - \beta(B)B + (\bar{\theta} + \beta(B) - \alpha(A))\tilde{V}. \tag{6}$$

This leads to the definition of an equilibrium.

Definition 1 *Given any liquidity shock $\hat{X}_h$, an equilibrium $(\Theta_h^*(A,B),\ \Theta_n^*(A,B),\ (A^*,B^*))$ is such that*[21]

1. *given any A and B, $\Theta_i^*(A,B)$ solves a type i investor's Problem (1) − (3) for $i \in \{h,n\}$;*

[19] To help remember, α (Alpha) denotes Ask depth and β (Beta) denotes Bid depth.

[20] One of the roles of a designated market-maker is to provide liquidity. As shown later, in our model the market-maker always trades when others have needs to trade and thus in this sense always provides liquidity. In addition, in Section 6.C, we also consider the case in which the market-maker faces an upper limit on the bid-ask spread she can charge.

[21] The market clearing conditions (Equation (4)) are implicitly enforced in the market-maker's problem.

2. given $\Theta_h^*(A, B)$ *and* $\Theta_n^*(A, B)$*,* A^* *and* B^* *solve the market-maker's Problem* $(5) - (6)$*.*

3.A Discussions on the assumptions of the model

In this subsection, we discuss our main assumptions and whether these assumptions are important for our main results.

The assumption that there is only one market-maker is for expositional simplicity. We have also solved a model with multiple market-makers. In this more general model with Cournot competition, we show that our main qualitative results still hold (e.g., short-sale constraints increase the expected bid-ask spread).[22] The assumption that the market-maker is risk-averse is not important for the main results that short-sale constraints decrease bid price and bid depth, but increase bid-ask spread. With a risk-neutral market-maker, the difference is that short-sale constraints no longer affect ask or ask depth.

The existing empirical analyses of how short-sale constraints affect spreads focus on the spread difference shortly after the constraint imposition dates. Accordingly, we use a one-period setting to examine the immediate impact of short-sale constraints. This one period setting also helps highlight the main driving forces behind our results and simplifies exposition. As we show in Section 6, extending to a dynamic model does not change the immediate impact of short-sale constraints. The assumption that the market-maker can make offsetting trades at bid and ask simultaneously is not critical for our main results. Even when the market-maker *cannot* make an offsetting trade, short-sale constraints still decrease bid and bid depth. This is because as we show later, the market-maker's market power in the bid market is the key driving force for the result.

To keep the exposition as simple as possible to show the key driving force behind our main results, we assume that there is no information asymmetry in the main model. We relax this assumption in Section 6 to demonstrate the robustness of our results to the presence of information asymmetry even with reduced information revelation due to short-sale constraints. We assume

[22]Market-makers in Cournot competition still retain some market power even when there are more than two market-makers. In contrast, as is well-known, it takes only two Bertrand competitors to reach a perfect competition equilibrium. However, market prices can be far from those of perfect competition (e.g., Christie and Schultz (1994), Chen and Ritter (2000), and Biais, Bisi‘ere and Spatt (2010)).

that the market-maker posts price schedules first and then other investors submit orders to the market-maker. This is consistent with the common practice in less competitive markets in which a designated market-maker making two-sided markets typically provides a take-it-or-leave-it pair of prices, a bid and an offer, to customers (e.g., Duffie (2012), Chapter 1).

In accordance with the existing literature on the impact of short-sale constraints, we do not explicitly take into account the possibility that the imposition of short-sale constraints itself may convey negative information about the stock payoff. However, the effect of this negative signal is clear from our model, i.e., it decreases both bid and ask prices. Therefore, while the result that short-sale constraints increase the ask price might be reversed if this negative information effect dominates, the main result that short-sale constraints decrease bid price would be strengthened. In addition, if the negative information effect lowers bid and ask by a similar amount, the result that short-sale constraints increase the spread would also likely hold. Moreover, empirical studies show that the increases in bid-ask spreads following short-sale bans are not driven by any negative information possibly conveyed by the impositions themselves (e.g., Beber and Pagano (2013), Boehmer, Jones and Zhang (2013), and Ang, Shtauber, and Tetlock (2013)).

4. The equilibrium

In this section, we solve for the equilibrium bid and ask prices, bid and ask depths, and trading volume in closed form.

Given A and B, the optimal demand schedule for a type i investor for $i \in \{h, n\}$ is

$$\theta_i^*(A,B) = \begin{cases} \frac{P_i^R - A}{\delta\sigma_V^2} & A < P_i^R, \\ 0 & B \le P_i^R \le A, \\ \max\left[-(\kappa_i + \bar{\theta}), -\frac{B - P_i^R}{\delta\sigma_V^2}\right] & B > P_i^R, \end{cases} \tag{7}$$

where

$$P_i^R = \bar{V} + \omega \hat{X}_i - \delta\sigma_V^2\bar{\theta} \tag{8}$$

is the reservation price of a type i investor (i.e., the critical price such that a non-market-maker buys (sells, respectively) the security if and only if the ask price is lower (the bid price is higher, respectively) than this critical price) and $\omega = -\delta\sigma_{VL}$ represents the hedging premium per unit of the liquidity shock.

Let Δ denote the difference between the reservation prices of h and n investors, i.e.,

$$\Delta := P_h^R - P_n^R = \omega \hat{X}_h. \tag{9}$$

The following theorem provides the equilibrium bid and ask prices and equilibrium security demand in closed form.

Theorem 1 *1. If* $-\frac{2(N+1)\delta\sigma_V^2(\kappa_h+\bar{\theta})}{N_n+2} < \Delta < \frac{2(N+1)\delta\sigma_V^2(\kappa_n+\bar{\theta})}{N_h}$*, then short-sale constraints do not bind for any investors,*

(a) the equilibrium bid and ask prices are

$$A^* = P_n^R + \frac{N_h}{2\,(N+1)}\Delta + \frac{\Delta^+}{2}, \tag{10}$$

$$B^* = P_n^R + \frac{N_h}{2\,(N+1)}\Delta - \frac{\Delta^-}{2}, \tag{11}$$

the bid-ask spread is

$$A^* - B^* = \frac{|\Delta|}{2} = \frac{|\omega \hat{X}_h|}{2}, \tag{12}$$

(b) the equilibrium quantities demanded are

$$\theta_h^* = \frac{N_n + 2}{2\,(N+1)\,\delta\sigma_V^2}\Delta, \quad \theta_n^* = -\frac{N_h}{2\,(N+1)\,\delta\sigma_V^2}\Delta, \quad \theta_m^* = 2\theta_n^*, \tag{13}$$

and the equilibrium quote depths are

$$\alpha^* = N_h(\theta_h^*)^+ + N_n(\theta_n^*)^+, \tag{14}$$

$$\beta^* = N_h(\theta_h^*)^- + N_n(\theta_n^*)^-; \tag{15}$$

2. *If $\Delta \leq -\frac{2(N+1)\delta\sigma_V^2(\kappa_h+\bar{\theta})}{N_n+2}$, then short-sale constraints bind for hedgers,*

(a) *the equilibrium bid and ask prices are*

$$A_{c1}^* = P_n^R - \frac{\delta N_h \sigma_V^2(\kappa_h + \bar{\theta})}{N_n + 2}, \tag{16}$$

$$B_{c1}^* = P_h^R + \delta\sigma_V^2(\kappa_h + \bar{\theta}), \tag{17}$$

the bid-ask spread is

$$A_{c1}^* - B_{c1}^* = -\Delta - \frac{N+1}{N_n+2}\delta\sigma_V^2(\kappa_h + \bar{\theta}), \tag{18}$$

(b) *the equilibrium quantities demanded are*

$$\theta_{hc1}^* = -(\kappa_h + \bar{\theta}), \quad \theta_{nc1}^* = \frac{N_h(\kappa_h + \bar{\theta})}{N_n + 2}, \ \theta_{mc1}^* = \frac{2N_h(\kappa_h + \bar{\theta})}{N_n + 2}, \tag{19}$$

and the equilibrium quote depths are

$$\alpha_{c1}^* = \frac{N_h N_n(\kappa_h + \bar{\theta})}{N_n + 2}, \ \beta_{c1}^* = N_h(\kappa_h + \bar{\theta}); \tag{20}$$

3. *If $\Delta \geq \frac{2(N+1)\delta\sigma_V^2(\kappa_n+\bar{\theta})}{N_h}$, then short-sale constraints bind for non-hedgers,*

(a) *the equilibrium bid and ask prices are*

$$A_{c2}^* = P_h^R - \frac{\Delta + \delta N_n \sigma_V^2(\kappa_n + \bar{\theta})}{N_h + 2}, \tag{21}$$

$$B_{c2}^* = P_n^R + \delta\sigma_V^2(\kappa_n + \bar{\theta}), \tag{22}$$

the bid-ask spread is

$$A_{c2}^* - B_{c2}^* = \frac{N_h+1}{N_h+2}\Delta - \frac{N+1}{N_h+2}\delta\sigma_V^2(\kappa_n + \bar{\theta}), \tag{23}$$

(b) the equilibrium quantities demanded are

$$\theta^*_{hc2} = \frac{\Delta + \delta N_n \sigma_V^2(\kappa_n + \bar{\theta})}{(N_h + 2)\delta\sigma_V^2}, \quad \theta^*_{nc2} = -(\kappa_n + \bar{\theta}), \tag{24}$$

$$\theta^*_{mc2} = \frac{-N_h\Delta + 2\delta N_n \sigma_V^2(\kappa_n + \bar{\theta})}{(N_h + 2)\delta\sigma_V^2}, \tag{25}$$

and the equilibrium quote depths are

$$\alpha^*_{c2} = \frac{N_h\Delta + \delta N_h N_n \sigma_V^2(\kappa_n + \bar{\theta})}{(N_h + 2)\delta\sigma_V^2}, \;\; \beta^*_{c2} = N_n(\kappa_n + \bar{\theta}). \tag{26}$$

Theorem 1 shows that whether short-sale constraints bind for some investors depends on whether the magnitude of the reservation price difference is sufficiently large. If hedgers' reservation price is close to that of non-hedgers, then no one trades a large amount in equilibrium, and thus short-sale constraints do not bind for any of the investors (Case 1). If hedgers' reservation price is much lower than that of non-hedgers, then the equilibrium bid price in the no-constraint case is much higher than the reservation price of hedgers, hedgers would like to sell a large amount, and thus short-sale constraints bind for hedgers (Case 2). The opposite is true if non-hedgers' reservation price is much lower than that of hedgers (Case 3). The thresholds for the reservation price difference such that short-sale constraints bind are determined by equalizing the unconstrained equilibrium short-sale quantities (θ^*_h or θ^*_n) to the short-sale bounds ($-(\kappa_h + \bar{\theta})$ or $-(\kappa_n + \bar{\theta})$ respectively).

Part 1 of Theorem 1 implies that when short-sale constraints do not bind, in equilibrium both bid and ask prices are nonlinear functions of the reservation prices of hedgers and non-hedgers. In addition, non-hedgers can indeed infer $\hat{X}_h$ from observing the equilibrium trading price as we conjectured, because of the one-to-one mapping between the two. Furthermore, Part 1 shows that, when short-sale constraints do not bind, the equilibrium bid-ask spread is equal to half of the absolute value of the reservation price difference between hedgers and non-hedgers.

When short-sale constraints bind for hedgers or non-hedgers, the maximum amount of purchase that the market-maker can make with the constrained investors is fixed, and thus the market-maker's utility always decreases in the bid price in the region in which the constraints bind. As explained in the next section, the market power of the market-maker then indicates that the optimal bid price when short-sale constraints bind in equilibrium must be such that short-sale constraints just start to bind, which gives rise to the constrained equilibrium bid prices as in (17) and (22), and bid depths as in (20) and (26). Given these bid prices and depths, ask prices and depths are then determined optimally by the market-maker to trade off profit from the spread and the inventory risk, taking into account the demand schedules of the buyers.

5. The effect of short-sale constraints

In this section, we analyze the effect of short-sale constraints on bid prices, ask prices, bid-ask spreads, and liquidity-risk.

5.A Bid/ask prices, bid-ask spread, and trading volume

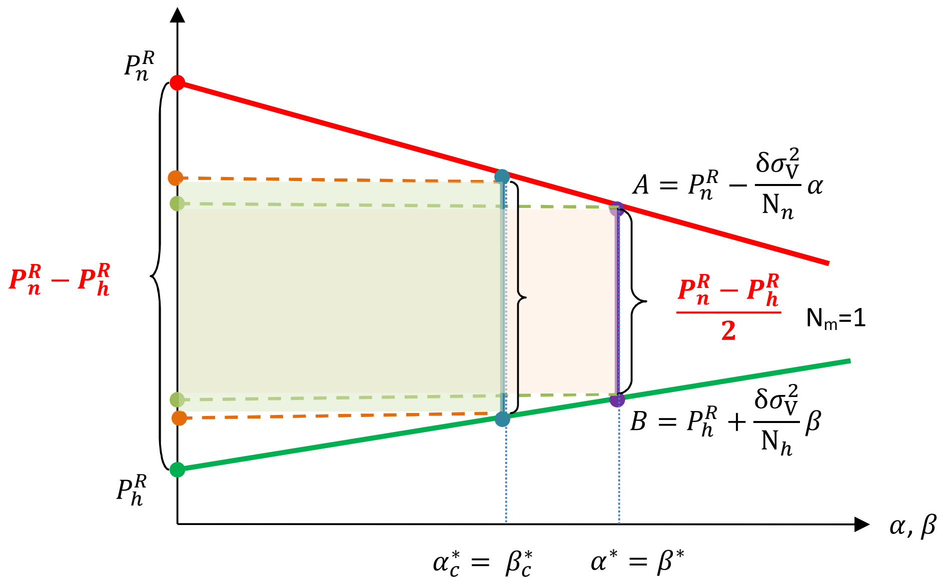
By Theorem 1, we have:

Proposition 1 *1. As short-sale constraints become more stringent for hedgers or non-hedgers, the equilibrium bid price decreases, the equilibrium ask price increases, and so does the equilibrium bid-ask spread.*

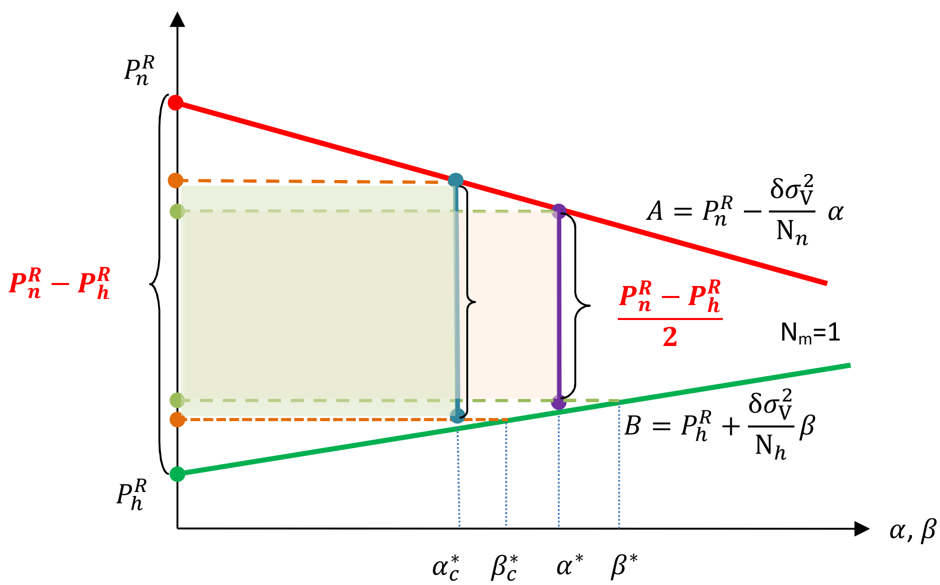
2. As short-sale constraints become more stringent for hedgers or non-hedgers, the equilibrium bid depth, the equilibrium ask depth, and the equilibrium trading volume decrease.

Because short-sale constraints restrict sales at the bid, one might expect that short-sale constraints increase the equilibrium bid price. In contrast, Proposition 1 implies that prohibition of short-sales decreases the bid. We next provide the essential intuition for this seemingly counter-intuitive result and other implications of Proposition 1 through graphical illustrations. Suppose $P_h^R < P_n^R$, and thus hedgers sell and non-hedgers buy in equilibrium. The market clearing condition (4) implies that the inverse demand and supply functions faced by the market-maker are, respectively,

$$A = P_n^R - \frac{\delta\sigma_V^2}{N_n}\alpha, \quad B = P_h^R + \frac{\delta\sigma_V^2}{N_h}\beta.$$



(a) Infinitely risk-averse market-maker with $\bar{\theta} = 0$



(b) Finitely risk-averse market-maker

Figure 1: Inverse demand/supply functions and bid/ask prices with and without short-sale constraints.

To make the intuition as simple as possible, we first plot the above inverse demand and supply functions and equilibrium spreads in Figure 1(a) for the extreme case in which the market-maker has infinite risk aversion and no initial endowment of the risky security.[23] Then we illustrate in Figure 1(b) the case in which the market-maker has the same risk-aversion and initial endowment as other investors. Figure 1 shows that, as the market-maker decreases bid (increases ask) other investors sell (buy) less. Facing the inverse supply and demand functions, a monopolistic market-maker optimally trades off profit from the spread and inventory risk. Similar to the results of monopolistic competition models, the bid and ask spread is equal to the absolute value of the reservation price difference $|\Delta|$, divided by 2 (by $N_m + 1$ with multiple market-makers engaging in Cournot competition). In Figure 1(a) because the market-maker has infinite risk-aversion and no initial endowment, the market-maker buys the same amount at the bid as the amount she sells at the ask, so that there is zero inventory carried to date 1. With short-sale constraints binding for hedgers, a market-maker can only buy from hedgers up to $N_h(\kappa_h + \bar{\theta})$, no matter how high the bid price is. Because the market-maker has market power and obtains a greater utility with a lower bid price when the amount of purchase at the bid is fixed, the market-maker chooses a lower bid price such that the short-sale constraint never strictly binds. Therefore, if the unconstrained equilibrium sale amount from hedgers is larger than the upper bound $N_h(\kappa_h + \bar{\theta})$ permitted by the short-sale constraints, the market-maker lowers the bid price such that in the constrained equilibrium, hedgers sell less and the short-sale constraints just start to bind. Because the market-maker buys less from hedgers in equilibrium, the market-maker must sell less to non-hedgers at the ask than in the unconstrained case to avoid inventory risk. Therefore, the market-maker optimally increases the ask price to achieve the desired reduced amount of sale. When the market-maker has positive but finite risk-aversion, the same motive of reducing inventory risk also drives up the ask price and drives down the ask depth, although the market-maker may choose to carry some inventory.

[23] Even though in the main model, we assume that the market-maker has the same risk-aversion as other investors, this extreme case can be easily solved to yield the results shown in this figure. In this special case, the ask depth is always equal to the bid depth and the market-maker maximizes only the profit from the spread and carries no inventory.

On the other hand, as shown in the following proposition, if the market-maker is risk-neutral, then short-sale constraints only reduce bid prices and bid depth, but do not affect ask prices or ask depth, because inventory risk is irrelevant for her.

Proposition 2 *For a risk-neutral market-maker, short-sale constraints still reduce bid prices and bid depth, but have no impact on ask prices or ask depth.*

As a result, even with a risk-neutral market-maker, short-sale constraints still increase bid-ask spreads. The above intuition suggests that position limits on long positions would have the same qualitative impact: increasing ask prices, decreasing bid prices, and thus increasing bid-ask spread; and decreasing bid and ask depths, thus also reducing trading volume.[24]

To further identify the driving force behind the reduction of bid price due to short-sale constraints, in Theorem 6 in Appendix B we report the equilibrium results for an alternative model in which the market-maker is a price-taker in the "bid" market as in most of the extant literature, but a monopolist in the "ask" market as in the main model. Theorem 6 shows the same qualitative results for the impact of short-sale constraints on the ask price, bid and ask depths, and trading volume. However, in contrast to the main model, Theorem 6 implies that short-sale constraints *increase* equilibrium bid price. Because this alternative model differs from our main model only in that the market-maker is a price-taker in the "bid" market, this shows that the driving force behind our result that short-sale constraints decrease bid price is indeed the market-maker's market power. If buyers do not have market power (i.e., are price-takers), then they compete for the reduced supply and thus the constrained equilibrium price becomes higher.

Our model predicts that in markets in which market-makers have market power and are risk-averse, imposing short-sale constraints will cause bid prices to go down and ask prices to go up. There is a caveat for this result: as in the existing literature, we do not model explicitly the information content of the imposition itself. The imposition of short-sale constraints by regulators may signal some negative information about the stocks being regulated. If this negative information content was taken into account, then the joint impact of this negative signal and short-sale

[24]The results on the effect of position limits (on both long and short positions) on prices and depths are available from the authors.

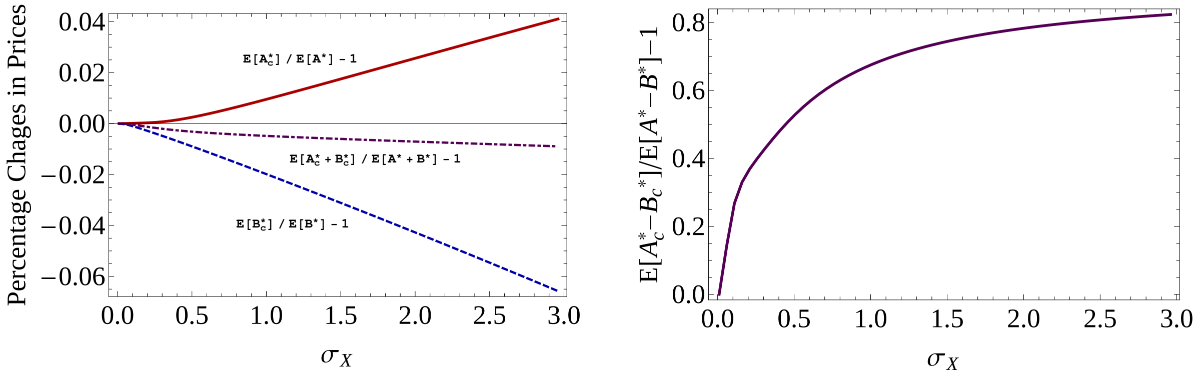


Figure 2: The percentage changes in the expected bid price (dashed), the expected ask price (solid), the expected mid-price (dot-dashed), and the expected bid-ask spread with short-sale constraints against σ_X. The parameter values are: $\delta = 1$, $\sigma_V = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta} = 1/(N_h + N_n + N_m)$, and $\kappa_h = \kappa_n = 0$.

constraints would lower the bid price further, but might also lower the ask price in the net. This is because negative information drives both bid price and ask price down, as implied by Theorem 1. On the other hand, because negative information drives both bid price and ask price down, the information content of the imposition of the constraints affects the result less that short-sale constraints increase bid-ask spread, as long as the magnitude of the impact on bid is similar to that on ask.[25]

To illustrate the average magnitude of the impact across all possible realizations of the liquidity shock $\hat{X}_h$, we plot the percentage changes in expected bid, expected ask, and expected spread in Figure 2 against the liquidity shock volatility σ_X. Figure 2 shows that short-sale prohibition can have significant impact on expected prices and especially on expected spread.[26] For example, the bid price can go down by more than 6% and the ask price can go up by more than 4%, even though on average there is no liquidity shock for hedgers.[27] With a liquidity shock volatility of 1.0, the average

[25]If one models the impact of the information content of short-sale constraints imposition as having a lower unconditional expected payoff $\bar{V}$ in the case with short-sale constraints than without, then in our model, equilibrium bid and ask prices decrease by the same amount and thus the spread would be unaffected.

[26]All of the figures in the paper are for illustrations of qualitative results only and we do not attempt to calibrate to imperfectly competitive markets. $\bar{\theta}$ is chosen to normalize the total supply of the security to 1 share.

[27]Figure 2 shows that the mid quote price can also go down with short-sale constraints, depending on the relative magnitudes of the elasticities of demand and supply.

spread increases by more than 60%. In addition, the impact of short-sale constraints increases with the liquidity shock volatility. Intuitively, as the liquidity shock volatility increases, not only the probability that short-sale constraints bind increases, but also conditional on constraints binding, the average impact on bid and ask prices increases. Consequently, the unconditional average impact becomes greater with a higher liquidity shock volatility.

5.B Bid-ask spread volatility

One type of liquidity-risk faced by investors just prior to time 0 is that time 0 bid-ask spread is stochastic, depending on realizations of liquidity shock $\hat{X}_h$. The more volatile the spread, the greater the risk.[28] We next study how short-sale constraints affect the time 0 volatility of bid-ask spread. To this end, we have

Proposition 3 *Short-sale constraints increase a stock's liquidity-risk measured by the time 0 bid-ask spread volatility, i.e.,* $Vol(A_c^* - B_c^*) \geq Vol(A^* - B^*)$.

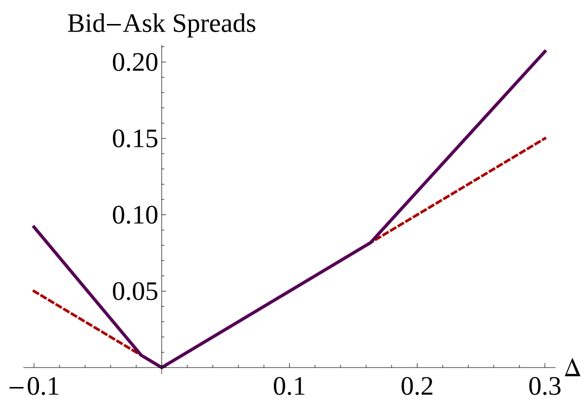


Figure 3: Spread as a function of reservation price difference Δ. The solid (dashed) lines represent the spreads with and without short-sale constraints respectively. The default parameters are: $\delta = 1$, $\sigma_V = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta} = 1/(N_h + N_n + N_m)$, and $\kappa_h = \kappa_n = 0$.

[28] Note that the time 1 bid-ask spread is zero, because the payoff becomes publicly known at time 1 and thus both bid and ask prices are equal to the payoff. This indicates that just prior to time 0, there is only uncertainty about the time 0 bid-ask spread, but no uncertainty about time 1 spread. Thus the time 0 spread volatility can also be interpreted as the volatility of the change in the spread between time 0 and time 1, i.e., time series volatility.

Thus our model predicts that after the imposition of short-sale constraints, the volatility of spread increases. The main intuition for Proposition 3 is as follows. When short-sale constraints bind, there is less risk-sharing among investors and thus bid and ask prices change more in response to a random shock. For example, keeping everything else constant, we plot the bid-ask spreads as a function of reservation price difference Δ for the case without constraints (dashed lines) and the case with short-sale constraints (solid lines) in Figure 3. This figure shows that indeed when the constraints bind, for the same change in the reservation price difference, spread changes more (i.e., steeper lines), which in turn implies that the volatility of spread goes up.

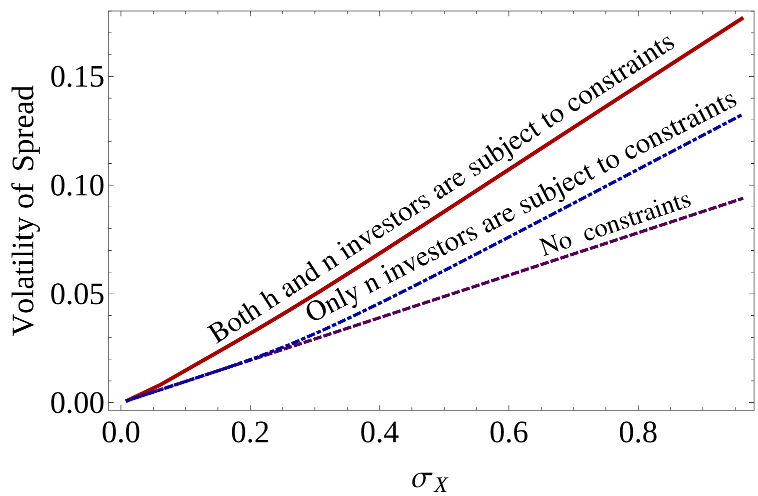


Figure 4: The volatility of bid-ask spread with and without short-sale constraints against σ_X. The default parameters are: $\delta = 1$, $\sigma_V = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta} = 1/(N_h + N_n + N_m)$, and $\kappa_h = \kappa_n = 0$.

To examine the magnitudes of the impact of short-sale constraints on the spread volatility, we plot bid-ask spread volatilities against liquidity shock volatility σ_X for three cases: 1) no one is subject to short-sale constraints (dashed); 2) only non-hedgers are subject to short-sale constraints (dot-dashed); and 3) both hedgers and non-hedgers are subject to short-sale constraints (solid). Consistent with Proposition 3, Figure 4 illustrates that spread volatility is the greatest when both hedgers and non-hedgers are subject to short-sale constraints and the lowest when no one is subject to short-sale constraints. The differences in spread volatilities across the three cases in Figure 4 suggest that the impact of short-sale constraints on spread volatility can be significant. For example, with a liquidity shock volatility of 1.0, the spread volatility in Case (3) can be more than double that in Case (1). In addition, as the liquidity shock volatility increases, this impact increases. This is because with a higher liquidity shock volatility, short-sale constraints bind more often and are also more restrictive on average conditional on binding.

6. Equilibrium models with information asymmetry, information acquisition, reduced information revelation, and dynamic trading

To highlight the key driving forces, we have so far assumed that there is no information asymmetry and there is only one trading period. In this section, to check the robustness of our results, we derive the equilibrium in the presence of information asymmetry (with and without reduced information revelation, with and without endogenous information acquisition), and extend the model to a dynamic setting.

6.A Equilibrium with asymmetric information

In this subsection, we extend our model to incorporate asymmetric information. To ensure that the private information about the risky security's payoff does not affect hedging-demand, we decompose the date 1 payoff $\tilde{V}$ of each share into $\tilde{v} + \tilde{u}$, where $\tilde{v} \sim \mathbf{N}(\bar{V}, \sigma_v^2)$ and $\tilde{u} \sim \mathbf{N}(0, \sigma_u^2)$ are independent with $\sigma_v > 0$, $\sigma_u > 0$, $\mathrm{Cov}(\tilde{u}, \tilde{L}) = \sigma_{VL}$, and $\mathrm{Cov}(\tilde{v}, \tilde{L}) = 0$.

We assume that on date 0, hedgers observe a private signal

$$\hat{s} = \tilde{v} - \bar{V} + \tilde{\varepsilon} \tag{27}$$

about the payoff $\tilde{v}$, where $\tilde{\varepsilon}$ is independently normally-distributed with mean zero and variance σ_ε^2.[29] Because the payoff of the nontraded asset $\tilde{L}$ is independent of the first component $\tilde{v}$ (i.e.,

[29] Observing the private signal may also be reinterpreted as extracting more precise information from public news

$\mathrm{Cov}(\tilde{v}, \tilde{L}) = 0$), private information about the security payoff does not affect the hedging-demand. Thus, for hedgers, information motivated trades are separated from hedging motivated trades.[30] Assuming it is hedgers who observe the private signal is to preserve information asymmetry in equilibrium.[31] Because hedgers have private information and non-hedgers do not, we will also refer to hedgers as the informed, and non-hedgers as the uninformed in this and subsequent extensions with asymmetric information.

To examine how information asymmetry affects the impact of short-sale constraints, we need a measure of information asymmetry. To this extent, we assume that there is a public signal

$$\hat{S}_s = \hat{s} + \hat{\eta} \tag{28}$$

about hedgers' private signal $\hat{s}$ that all investors (i.e., non-hedgers, the designated market-maker, and hedgers) can observe, where $\hat{\eta}$ is independently normally distributed with mean zero and volatility $\sigma_\eta > 0$. This public signal represents public disclosure about the asset payoff determinants, such as macroeconomic conditions, cash flow news, and regulation shocks, which is correlated with but less precise than hedgers' private signal. As demonstrated in Liu and Wang (2016), the volatility σ_η can serve as a clean measure of information asymmetry which does not affect aggregate information quality in the economy (measured by the precision of security payoff distribution conditional on *all* information in the economy).[32]

Because investors of type $i \in \{h, n, m\}$ are ex ante identical, we restrict our analysis to symmetric equilibria in which all investors of the same type adopt the same trading strategy. Investors' problems are exactly the same as those in the main model, except that their information sets are different. Let $\mathcal{I}_i$ represent a type i investor's information set on date 0 for $i \in \{h, n, m\}$. Because

than the uninformed (e.g., Engelberg, Reed, and Ringgenberg (2012)).

[30]This way, hedgers' trades can also be viewed as pooled trades from pure information traders and pure liquidity traders.

[31]If it were non-hedgers who observe the private signal, then because hedgers know their own liquidity shock, they would be able to infer the private signal precisely from equilibrium prices and thus there would be no information asymmetry in equilibrium.

[32]For example, the precision of a private signal about the risky security payoff would not be a good measure of information asymmetry, because a change in the precision also changes the quality of aggregate information about the payoff and both information asymmetry and information quality can affect economic variables of interest (e.g., prices, liquidity).

hedgers know exactly $\{\hat{s}, \hat{X}_h\}$, we have $\mathcal{I}_h = \{\hat{s}, \hat{X}_h\}$,

$$\mathrm{E}[\tilde{V}|\mathcal{I}_h] = \bar{V} + \rho_h \hat{s}, \quad \mathrm{Var}[\tilde{V}|\mathcal{I}_h] = (1-\rho_h)\sigma_v^2 + \sigma_u^2, \tag{29}$$

where

$$\rho_h := \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}. \tag{30}$$

The hedgers' reservation price becomes

$$P_h^R = \bar{V} + \hat{S} - \delta((1-\rho_h)\sigma_v^2 + \sigma_u^2)\bar{\theta}, \tag{31}$$

where $\hat{S} := \rho_h \hat{s} + \omega \hat{X}_h$.

Given that the joint impact of $\hat{s}$ and $\hat{X}_h$ on hedgers' demand is through the composite signal $\hat{S}$, we restrict our analysis to equilibrium prices A^* and B^* that are piecewise linear in the composite signal $\hat{S}$ and the public signal $\hat{S}_s$ and conjecture that other investors can infer the value of $\hat{S}$ (but not $\hat{s}$) from the realized market prices.[33] Accordingly, the information sets for the non-hedgers and the market-maker are

$$\mathcal{I}_n = \mathcal{I}_m = \{\hat{S}, \hat{S}_s\}. \tag{32}$$

Then the conditional expectation and conditional variance of $\tilde{V}$ for non-hedgers and the market-maker are respectively

$$E[\tilde{V}|\mathcal{I}_n] = \bar{V} + \rho_n(1-\rho_X)\hat{S} + \rho_n \rho_X \rho_h \hat{S}_s, \tag{33}$$

$$\mathrm{Var}[\tilde{V}|\mathcal{I}_n] = (1-\rho_n\rho_h)\sigma_v^2 + \sigma_u^2, \tag{34}$$

where

$$\rho_X := \frac{\omega^2 \sigma_X^2}{\omega^2 \sigma_X^2 + \rho_h^2 \sigma_\eta^2}, \quad \rho_n := \frac{\sigma_v^2}{\sigma_v^2 + \rho_X \rho_h \sigma_\eta^2}. \tag{35}$$

[33] Given that the market-maker takes into account the best response of other investors in the posted price schedules, the equilibrium trading quantities (equivalently the realized prices implied by these quantities) reveal $\hat{S}$.

It follows that the reservation prices for the non-hedgers and the market-maker are equal to

$$P_n^R = P_m^R = \bar{V} + \rho_n(1 - \rho_X)\hat{S} + \rho_n\rho_X\rho_h\hat{S}_s - \delta((1 - \rho_n\rho_h)\sigma_v^2 + \sigma_u^2)\bar{\theta}. \tag{36}$$

We solve this model with information asymmetry and report the equilibrium results in Theorem 4 in Appendix A, of which Theorem 1 is a special case with $\sigma_\varepsilon = \infty$. Theorem 4 shows that information asymmetry quantitatively changes the prices and quantities, but qualitative results remain the same. For example, when short-sale constraints do not bind, the spread is still equal to half of the absolute value of the reservation price difference between hedgers and non-hedgers. When short-sale constraints bind, the bid price is still such that the constraints just start to bind. Theorem 4 also shows that in equilibrium either the short-sale constraints do not bind or just start to bind and thus the trading quantity reveals the composite signal $\hat{S}$, consistent with our conjecture. In addition, in contrast to the perfect competition case or the case in which the market-maker is restricted to set $A = B$ as we consider in Section 6.C.2, there is no equilibrium where the composite signal $\hat{S}$ is not fully revealed. This is because (1) as argued previously, it is suboptimal for the market-maker to set a bid price such that the short-sale constraints strictly bind; and (2) if the equilibrium price without short-sale constraints would make the short-sale constraints strictly bind, then a bid price that is lower than the threshold price at which the short-sale constraints start to bind would make the short-sale constraints not binding, and the market-maker can be better off by increasing the bid price so that she can buy more from the sellers. This result demonstrates that the market power that can separate the bid market from the ask market may help improve the informativeness of market prices in the presence of short-sale constraints.

More importantly, we show in Appendix A that Proposition 1 holds with information asymmetry. Proposition 1 suggests that as long as short-sale constraints become more stringent for some investors, the bid price and depths decrease, but the ask price and spread increase. In particular, if some investors (e.g., the uninformed) cannot short-sell (possibly because of high short-sale costs) before a short-sale ban, then the imposition of the short-sale ban that prevents other investors (e.g., the informed) from shorting will make the bid price and depths decrease, but the ask price and spread increase. This is in sharp contrast with the conclusions of DV. To facilitate future empirical analysis, we next compare the predictions of our model with information asymmetry to those of DV using three main cases: Case 1: no short-sale constraints for any investors (i.e., $\kappa_n = \kappa_h = \infty$); Case 2: only the informed can short and without constraints (i.e., $\kappa_n = 0$, $\kappa_h = \infty$); and Case 3: short-sale prohibition for both the informed and the uninformed (i.e., $\kappa_n = \kappa_h = 0$). Case 2 is motivated by empirical evidence that short-sale costs can be smaller for relatively-informed investors and thus short-sellers tend to be more informed (e.g., Boehmer, Jones and Zhang (2008)). Proposition 1 implies that, in our model, whenever short-sale constraints are imposed on additional investors (Case 1 to Case 2 or Case 2 to Case 3), whether informed or uninformed, the expected bid price goes down, while the expected ask price and spread go up. In contrast, in DV, whether the informed or the uninformed become constrained is critical for their prediction. First, in contrast to our model, DV predict that immediately after a change from Case 1 to Case 3, neither the bid nor the ask changes and thus the spread also stays the same (see Corollary 2 in DV). The intuition in DV is that since short-sale prohibition restricts both the informed and the uninformed symmetrically, conditional on a sell order, the percentage of the informed-trading does not change and thus the conditional expected payoff remains the same. Because for a risk-neutral, competitive market-maker, the bid price is equal to the conditional expected payoff, the equilibrium bid price also remains the same. In addition, since the ask price is equal to the expected payoff conditional on a *buy* order and short-sale prohibition does not affect an investor's purchasing decision in their model, the ask price also remains the same. Second, consider a change from Case 2 to Case 3. Because the ban prohibits the informed from shorting, and thus a sell order becomes less likely from the informed, the DV model implies that in these markets the ban increases the expected bid price. As explained above, in the DV model, short-sale constraints do not have any impact on the ask price. This indicates that, as Boehmer, Jones and Zhang (2013) pointed out, the DV model predicts that the expected spread will go down after the additional short-sale ban on the informed. The main driving forces for the stark difference between the conclusions of these two models are the market power and the risk-aversion of the market-maker in our model. We summarize the main differences in predictions in Table 1. One can use these differences in predictions to test which

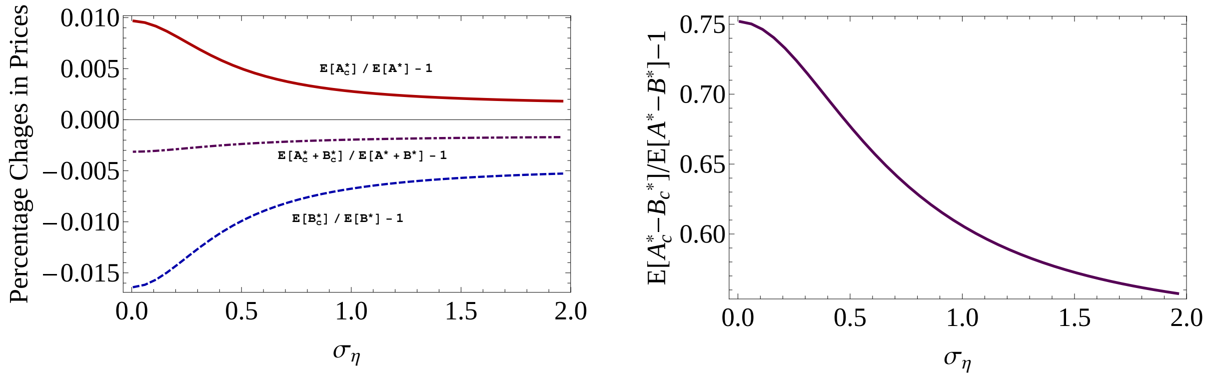


Figure 5: The percentage changes in the expected bid price (dashed), the expected ask price (solid), the expected mid-price (dot-dashed), and the expected bid-ask spread with short-sale constraints against σ_η. The parameter values are: $\delta = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $\sigma_X = 0.8$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta} = 1/(N_h + N_n + N_m)$, and $\kappa_h = \kappa_n = 0$.

theory applies better in which markets.

Table 1: Comparison of predictions on average bid, ask, and spread.

	This Paper			Diamond and Verrecchia (1987)		
Changes	Bid	Ask	Spread	Bid	Ask	Spread
Case 1 to Case 3	↓	↑	↑	—	—	—
Case 2 to Case 3	↓	↑	↑	↑	—	↓

Case 1: both unconstrained ($\kappa_n = \kappa_h = \infty$); Case 2: only the informed can short and without constraints ($\kappa_n = 0$, $\kappa_h = \infty$); Case 3: short-sale prohibition for both hedgers and non-hedgers ($\kappa_n = \kappa_h = 0$).

To illustrate the magnitudes of the impact of information asymmetry, we plot the percentage changes in expected bid, expected ask, and expected spread against the information asymmetry measure σ_η in Figure 5. Figure 5 shows that indeed short-sale constraints always decrease expected bid and increase expected ask even in the presence of asymmetric information. In addition, as information asymmetry increases, the magnitudes of the percentage changes in the expected ask, the expected bid, and the expected spread can all decrease. This is because as information asymmetry increases, both the adverse selection effect and the uncertainty faced by the uninformed increase.

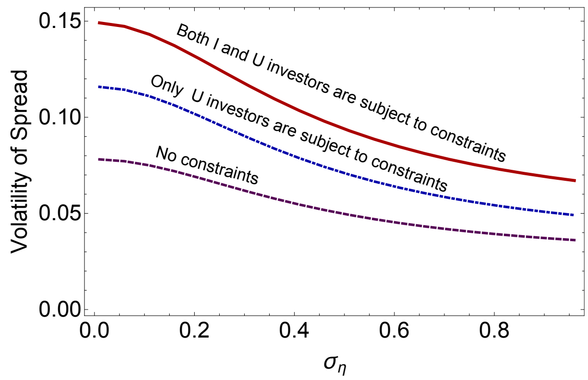


Figure 6: The volatility of the bid-ask spread with and without short-sale constraints against σ_η. The default parameter values are: $\delta = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $\sigma_X = 0.8$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\sigma_\eta = 0.4$, $\bar{\theta} = 1/(N_h + N_n + N_m)$, and $\kappa_h = \kappa_n = 0$.

Consequently investors may trade less on average, which results in the constraints binding less. Thus, our model predicts that ceteris paribus, the impact of short-sale constraints is greater for stocks with less information asymmetry.

To examine how information asymmetry affects the impact of short-sale constraints on spread volatility, we plot bid-ask spread volatilities against the information asymmetry measure σ_η in Figure 6 for the case without short-sale constraints (dashed), the case in which only uninformed are subject to short-sale constraints (dot-dashed), and the case in which both investors are subject to short-sale constraints (solid). As illustrated in Figure 6, spread volatility always increases as a result of short-sale constraints, even in the presence of asymmetric information. In addition, as information asymmetry increases, uncertainty increases, investors trade less, and thus the volatility increase caused by the short-sale constraints decreases. Thus our model also predicts that more public disclosure and greater transparency, which reduce information asymmetry, can increase the impact of short-sale constraints on liquidity-risk measured by the volatility of bid-ask spread.

6.B Endogenous information acquisition

We next examine whether our results can still hold when aggregate information quality is affected by imposing short-sale constraints. To this extent, we assume that, on date 0, the informed can acquire a costly signal $\hat{s}$ as defined in (27) with precision of $\rho_\varepsilon = \frac{1}{\sigma_\varepsilon^2}$ at a cost of $c(\rho_\varepsilon) := k\rho_\varepsilon^2$, where k is a positive constant.

For a given precision ρ_ε, we solve for the equilibrium prices and quantities as previously. We then solve for the optimal precision with and without short-sale constraints. We find that the optimal precision of private information for the informed in the presence of short-sale constraints tends to be lower than that in the absence of short-sale constraints, as shown in the upper panel of Figure 7. Intuitively, the presence of short-sale constraints may reduce the incentive of investors to acquire more precise information because short-sale constraints prevent them from fully benefiting from the private information in some states. Interestingly, Figure 7 illustrates that more public disclosure (i.e., smaller σ_η) might actually increase the incentive of the informed to acquire more precise private information. This is because public disclosure reduces information asymmetry and thus the loss of the informed from the adverse selection problem decreases. Figure 7 also suggests that the optimal precision increases with liquidity shock volatility. Intuitively, high liquidity shock volatility tends to increase the informed's trading volume and thus make them benefit more from more precise information.

More importantly, the lower panels of Figure 7 show that short-sale constraints may still decrease the expected bid price and increase the expected ask price, and the spread volatility even with endogenous information acquisition. In addition, as in the case with exogenous information acquisition, as the liquidity shock volatility increases, the impact of short-sale constraints increases, while as the information asymmetry increases, the impact tends to decrease. For a large set of parameter values, we obtain similar patterns to those shown in Figure 7. This demonstrates that our main results still hold even with endogenous information acquisition.

6.C Model extensions with more heterogeneity and reduced information revelation

In the above model with asymmetric information, all of the informed (i.e., hedgers) have the same information and in equilibrium all submit orders that reveal the composite signal $\hat{S}$. In addition, there is no restriction on the width of the spread that the market-maker can choose. We now extend

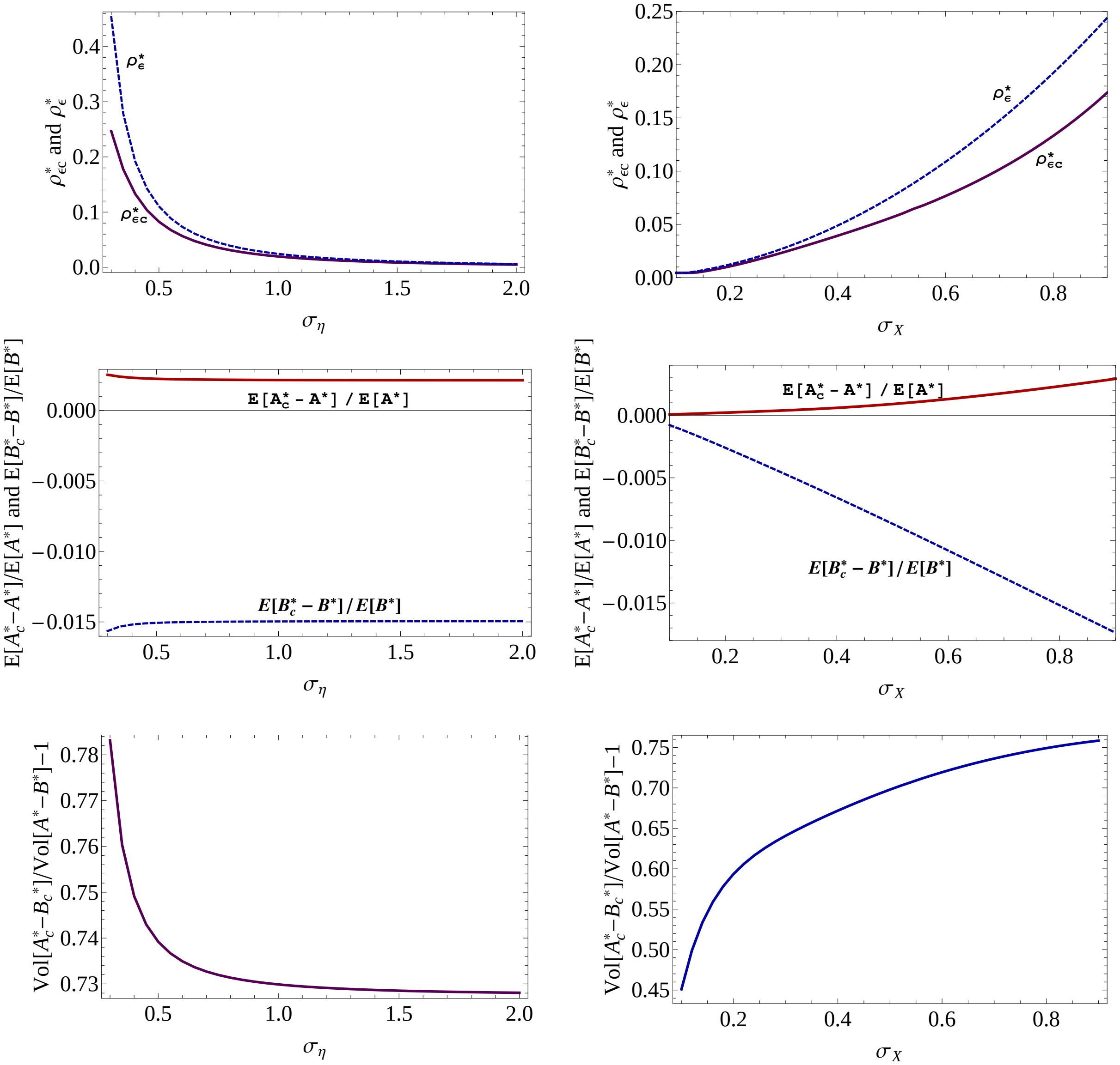


Figure 7: The optimal precision of information with and without short-sale constraints, and the percentage changes of expected bid, expected ask, and the spread volatility. The default parameters are: $\delta = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $\sigma_X = 0.8$, $N_h = 1$, $N_m = 1$, $N_n = 10$, $\bar{\theta} = 0.01$, $\kappa_h = \kappa_n = 0$, and $k = 0.001$.

our model to include multiple informed investors with different private information and their orders might not fully reveal the composite signal $\hat{S}$. We also allow initial endowment of the risky security, liquidity shocks, and risk-aversions to differ across investors. To model the fact that designated market-makers may face a regulatory cap on the spread that they can charge, and thus may have reduced market power, we consider the extreme case in which the spread is restricted to zero, i.e., bid must be equal to ask. Note that the market-maker still has (although reduced) market power even with the spread restricted to 0. This is because other investors still must trade through her, and thus she chooses the optimal price to maximize her expected utility taking into account how her choice affects the demand of other investors.

6.C.1 Robustness with more heterogeneity

Let $\bar{\theta}_i$, δ_i, $\hat{X}_i$, $\tilde{V}_i$ and $\mathcal{I}_i$ denote respectively the initial inventory, the risk-aversion coefficient, the liquidity shock, the date 1 resale value of the security, and the information set for a type i investor for $i \in \{h, n, m\}$. Given price P, for $i \in \{h, n\}$, a type i investor's problem is to choose θ_i to solve

$$\max E[-e^{-\delta_i \tilde{W}_i} | \mathcal{I}_i], \tag{37}$$

subject to the budget constraint

$$\tilde{W}_i = -\theta_i P + (\bar{\theta}_i + \theta_i)\tilde{V}_i + \hat{X}_i \tilde{L}, \tag{38}$$

and the short-sale constraint

$$\theta_i + \bar{\theta}_i \geq -\kappa_i. \tag{39}$$

By the same argument as previously, a type i investor's reservation price can be written as

$$P_i^R = E[\tilde{V}_i | \mathcal{I}_i] - \delta_i \mathrm{Cov}[\tilde{V}_i, \tilde{L} | \mathcal{I}_i]\hat{X}_i - \delta_i \mathrm{Var}[\tilde{V}_i | \mathcal{I}_i]\bar{\theta}_i, \quad i \in \{h, n, m\}. \tag{40}$$

The designated market-maker's problem is to choose price P to solve

$$\max E\left[-e^{-\delta_m \tilde{W}_m}|\mathcal{I}_m\right], \tag{41}$$

subject to

$$\tilde{W}_m = -\sum_{i=h,n} \min\left[\frac{P - P_i^R}{\delta_i \mathrm{Var}[\tilde{V}_i|\mathcal{I}_i]}, \kappa_i + \bar{\theta}_i\right] P + \left(\bar{\theta}_m + \sum_{i=h,n} \min\left[\frac{P - P_i^R}{\delta_i \mathrm{Var}[\tilde{V}_i|\mathcal{I}_i]}, \kappa_i + \bar{\theta}_i\right]\right) \tilde{V}_m. \tag{42}$$

Let $\Delta_{ij} := P_i^R - P_j^R$ denote the reservation price difference between Type i and Type j investors for $i, j \in \{h, n, m\}$. Define

$$\bar{P}_h = P_h^R + \delta_h(\kappa_h + \bar{\theta}_h)\mathrm{Var}[\tilde{V}_h|\mathcal{I}_h], \bar{P}_n = P_n^R + \delta_n(\kappa_n + \bar{\theta}_n)\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n], \tag{43}$$

$$P^* = \varphi_m P_m^R + \varphi_n P_n^R + (1 - \varphi_m - \varphi_n)P_h^R, \tag{44}$$

$$P_c^h = \lambda_m P_m^R + (1 - \lambda_m)P_n^R - \lambda_h \delta_h(\kappa_h + \bar{\theta}_h)\mathrm{Var}[\tilde{V}_h|\mathcal{I}_h], \tag{45}$$

$$P_c^n = \gamma_m P_m^R + (1 - \gamma_m)P_h^R - \gamma_n \delta_n(\kappa_n + \bar{\theta}_n)\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n], \tag{46}$$

where φ_m, φ_n, λ_m, λ_h, γ_m, and γ_n are as defined in (A-30)-(A-33) in the Appendix. $\bar{P}_h$ ($\bar{P}_n$) is the critical price above which short-sale constraints bind for h (n) investors, P^* is the equilibrium price in the absence of short-sale constraints, and P_c^h (P_c^n) is the equilibrium price given that hedgers (non-hedgers) short-sell κ_h (κ_n). In addition, let $V(P)$ denote the market-maker's expected utility in the absence of short-sale constraints and $V_c^h(P)$ be the market-maker's expected utility given that hedgers short-sell κ_h. For this alternative model with reduced market power, we report the results for the case with $\bar{P}_h \leq \bar{P}_n$ in Theorem 2.[34]

Theorem 2 *Suppose $\bar{P}_h \leq \bar{P}_n$.*

1. *If $P^* \leq \bar{P}_h$ and $P_c^h \leq \bar{P}_h$, short-sale constraints do not bind for any investor, and the equilibrium price is $P_c^* = P^*$;*

[34]The case $\bar{P}_h \geq \bar{P}_n$ is symmetric (i.e., switching "h" and "n" in the notations in Theorem 2) and thus omitted to save space.

2. *If $P^* \leq \bar{P}_h$ and $\bar{P}_h < P_c^h \leq \bar{P}_n$,*[35] *then the equilibrium price is*

$$P_c^* = \begin{cases} P^* & \text{if } V(P^*) > V_c^h(P_c^h), \\ P^* \text{ or } P_c^h & \text{if } V(P^*) = V_c^h(P_c^h), \\ P_c^h \ (> P^*) & \text{if } V(P^*) < V_c^h(P_c^h), \end{cases} \tag{47}$$

and if the equilibrium price is equal to P_c^h, then short-sale constraints bind for hedgers;

3. *If $P^* > \bar{P}_h \geq P_c^h$, then short-sale constraints bind for hedgers, and the equilibrium price is $P_c^* = \bar{P}_h \ (< P^*)$;*

4. *If $P^* > P_c^h > \bar{P}_h$ and $P_c^h \leq \bar{P}_n$, then short-sale constraints bind for hedgers, and the equilibrium price is $P_c^* = P_c^h \ (< P^*)$;*

5. *If $\bar{P}_n \geq P_c^h \geq P^* > \bar{P}_h$, then short-sale constraints bind for hedgers, and the equilibrium price is $P_c^* = P_c^h \ (\geq P^*)$;*

6. *If $P_c^h > \bar{P}_n > P^* > \bar{P}_h$, then short-sale constraints bind for both hedgers and non-hedgers, and the equilibrium price is $P_c^* = \bar{P}_n \ (> P^*)$;*

7. *If $P^* \geq \bar{P}_n$ and $P_c^h > \bar{P}_n$, then short-sale constraints bind for both hedgers and non-hedgers, and the equilibrium price is $P_c^* = \bar{P}_n \ (\leq P^*)$.*

It can be demonstrated that as the reservation price of the market-maker varies from low to high, all seven cases in Theorem 2 can occur. It is clear from Equations (43) through (46) that $P^* - \bar{P}_h$ and $P_c^h - \bar{P}_h$ both increase with P_m^R. The intuition for Theorem 2 is that if the reservation price of the market-maker is small enough relative to those of hedgers and non-hedgers, then the market-maker will be the only seller in equilibrium and thus short-sale constraints do not bind for any investor and the equilibrium price is the same as the case without short-sale constraints (Case 1). In the other extreme, if the reservation price of the market-maker is sufficiently large, then the market-maker is the only buyer and short-sale constraints bind for both hedgers and non-hedgers, which implies that the equilibrium price is equal to $\bar{P}_n$.

Cases 3, 4, and 7 reveal that the equilibrium price can still be lower with short-sale constraints even when the market-maker has a weaker market power. This suggests that for our results to hold the market power does not need to be extremely high.

[35] It can be shown that $P^* \leq \bar{P}_h$ and $P_c^h > \bar{P}_n$ cannot occur under the assumption that $\bar{P}_h \leq \bar{P}_n$.

On the other hand, Theorem 2 implies that the equilibrium price with short-sale constraints can be greater than the equilibrium price without the constraints when buyers have insufficient market power. For example, in Case 5 short-sale constraints bind for hedgers. If non-hedgers' reservation price is sufficiently high, then non-hedgers are buyers and the market-maker is a seller in equilibrium. In this case, short-sale constraints reduce supply, and the equilibrium price becomes higher because buyers (i.e., non-hedgers) do not have any market power, as in the extant literature that focuses on a competitive setting.[36]

6.C.2 Robustness with reduced information revelation

Theorem 4 implies that when the market-maker can separate bid and ask markets, i.e., charge a positive spread, the only equilibrium is the one in which the market prices fully reveal the composite signal $\hat{S}$. Next, we demonstrate that even when the market-maker cannot charge a positive spread and short-sale constraints reduce information revelation, the average equilibrium sale (bid) price with short-sale constraints may still be lower than that without the constraints. For this purpose, we consider the model in Subsubsection 6.C.1, specializing to the case in which hedgers (i.e., the informed) have zero initial endowment and non-hedgers (i.e., the uninformed) have different amount of initial endowment from that of the market-maker. As shown in Theorem 4 in Appendix A, the informed's demand increases with the composite signal $\hat{S}$ which combines the hedging-demand and information-motivated demand. When the composite signal $\hat{S}$ is smaller than a threshold $\underline{\mathrm{S}}$, the informed would like to short sell, but a no-short-sale constraint prevents such trading. In contrast to the model in Subsubsection 6.C.1, we assume that the informed do not submit any order in this case, and thus do not reveal the value of $\hat{S}$. The uninformed and the market-maker accordingly update their beliefs, conditional on the composite signal $\hat{S} < \underline{\mathrm{S}}$, and in this sense, information revelation is reduced by the presence of short-sale constraints. We provide the analysis details of this case at the end of Appendix A.

Figure 8 shows that the equilibrium price is a constant for all $\hat{S} < \underline{\mathrm{S}}$ and there is a discontinuous movement downward at $\hat{S} = \underline{\mathrm{S}}$. In other words, when the uninformed and the market-maker only know that $\hat{S} < \underline{\mathrm{S}}$, they use the conditional average of $\hat{S}$ for the estimation of $\hat{S}$, and therefore they underestimate $\hat{S}$ for $\underline{\underline{S}} < \hat{S} < \underline{\mathrm{S}}$, which is reflected by the downward discontinuity at $\underline{\mathrm{S}}$. On the other hand, they overestimate $\hat{S}$

[36]In addition, it is also possible that the equilibrium trade price becomes higher with the constraints than without, even when the market-maker is a buyer in equilibrium. For example, in Case 6, both hedgers and non-hedgers are sellers and constrained, and thus the market-maker is the buyer. However, Theorem 2 implies that in this case, the equilibrium price with the constraints is higher than that without. This occurs because the constraints reduce the amount that the market-maker can buy from the hedgers who are constrained at P^*, and the benefit of buying more from the non-hedgers outweighs the cost of a higher price than P^*.

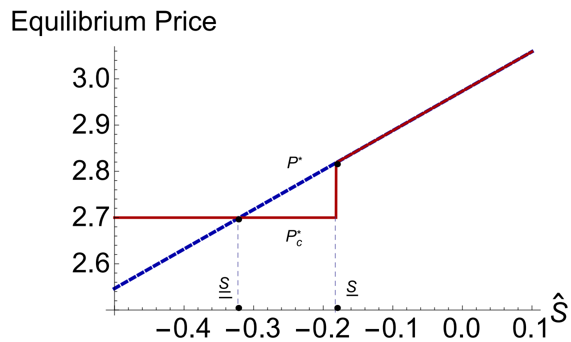


Figure 8: The unconstrained equilibrium price P^* and the constrained equilibrium price P_c^* against $\hat{S}$. The default parameters are: $\delta = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.4$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $\sigma_X = 0.3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta}_n = 0.1$, $\bar{\theta}_m = 0.6$, and $\kappa_h = \kappa_n = 0$.

for $\hat{S} < \underline{\underline{S}}$, as shown in Figure 8.

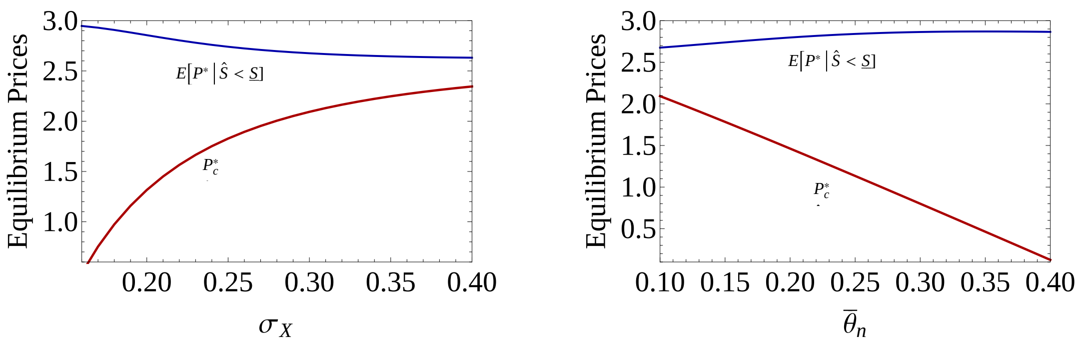


Figure 9: The expected unconstrained equilibrium price P^* and the constrained equilibrium price P_c^* conditional on $\hat{S} < \underline{S}$ against σ_X and $\bar{\theta}_n$ when the market-maker sells in equilibrium. The default parameters are: $\delta = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.4$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $\sigma_X = 0.3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta}_n = 0.1$, $\bar{\theta}_m = 0.6$, and $\kappa_h = \kappa_n = 0$.

Figure 9, in which the market-maker sells in equilibrium, and Figure 10, in which the market-maker buys in equilibrium, imply that even when short-sale constraints prevent some of the information of the informed from being revealed, the expected trading price with short-sale constraints can still be lower than that without the constraints. This result is true for a large set of parameter values. Intuitively, because the uninformed and the market-maker underestimate $\hat{S}$ for $\underline{\underline{S}} < \hat{S} < \underline{S}$, but overestimate $\hat{S}$ for $\hat{S} < \underline{\underline{S}}$, this translates to a lower equilibrium price for $\underline{\underline{S}} < \hat{S} < \underline{S}$, but a higher equilibrium price for $\hat{S} < \underline{\underline{S}}$ than the

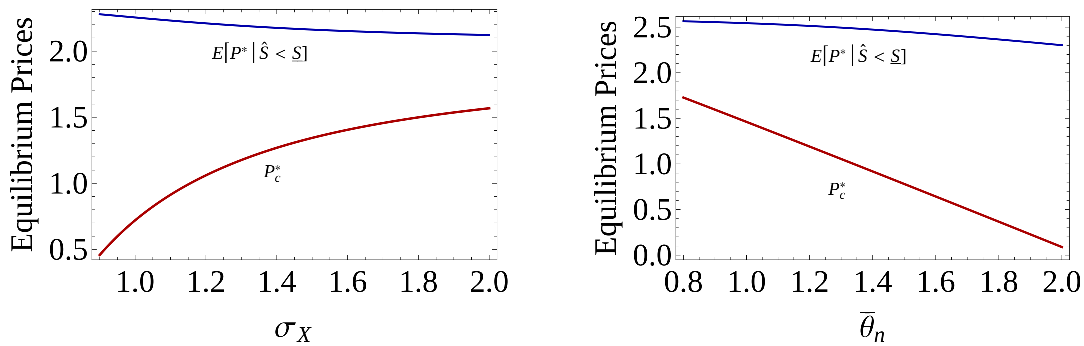


Figure 10: The expected unconstrained equilibrium price P^* and the constrained equilibrium price P_c^* conditional on $\hat{S} < \underline{S}$ against σ_X and $\bar{\theta}_n$ when the market-maker buys in equilibrium. The default parameters are: $\delta = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.4$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $\sigma_X = 0.3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta}_n = 2$, $\bar{\theta}_m = 0.2$, and $\kappa_h = \kappa_n = 0$.

unconstrained equilibrium price, as shown in Figure 8. Therefore, as long as the probability of $\underline{\underline{S}} < \hat{S} < \underline{\mathrm{S}}$ is significantly higher than the probability of $\hat{S} < \underline{\underline{S}}$, the equilibrium price with short-sale constraints for $\hat{S} < \underline{\mathrm{S}}$ is lower than the expected price without the constraints, conditional on $\hat{S} < \underline{\mathrm{S}}$. Because the equilibrium prices are the same with and without constraints for $\hat{S} \geq \underline{\mathrm{S}}$, the (unconditional) expected equilibrium price may also be lower with short-sale constraints.

6.D Robustness to a dynamic setting

One concern with our main model is that it is static. In this subsection, we demonstarte that it is likely that our main results still hold in a dynamic setting. For this purpose, we consider a two-period setting with trading dates 0, 1, and 2. Hedgers, non-hedgers, and the market-maker can trade a risk-free asset and a risky security on date 0 and 1 to maximize their CARA utility from the terminal wealth on date 2. The date 2 payoff of each share is $\tilde{V} + \tilde{\mu}$, where $\tilde{V}$ is observable on date 1, and $\tilde{\mu}$ is observable on date 2, $\tilde{V}$ and $\tilde{\mu}$ are independent, $\tilde{V} \sim \mathbf{N}(\bar{V}, \sigma_V^2)$, $\tilde{\mu} \sim \mathbf{N}(0, \sigma_\mu^2)$, $\bar{V}$ is a constant, $\sigma_V > 0$, $\sigma_\mu > 0$, and $\mathbf{N}(\cdot)$ denotes a normal distribution.

As previously, hedgers are subject to a liquidity shock that is modeled as a random endowment of $\hat{X}_h \sim \mathbf{N}(0, \sigma_X^2)$ units of a non-tradable risky asset on date 0. The non-traded asset has a per-unit payoff of $\tilde{L} + \tilde{e}$, where $\tilde{L} \sim \mathbf{N}(0, \sigma_L^2)$ has a covariance of $\sigma_{VL} > 0$ with $\tilde{V}$ and becomes public on date 1, $\tilde{e} \sim \mathbf{N}(0, \sigma_e^2)$ has a covariance of $\sigma_{\mu e} > 0$ with $\tilde{\mu}$ and becomes public on date 2, and $\tilde{L}$ and $\tilde{e}$ are independent.

Let θ_{it} denote the number of shares that an investor holds in the stock immediately after date t, $t = 0, 1$.[37] We assume that neither hedgers nor non-hedgers can short-sell, i.e.,

$$\theta_{it} \geq 0, \quad i = h, n, \ t = 0, 1. \tag{48}$$

Let P_t be the stock price on date t. For $i \in \{h, n\}$, investor i's problem is

$$\max_{\theta_{i0}, \theta_{i1}} E[-e^{-\delta \tilde{W}_{i2}}], \tag{49}$$

subject to the budget constraints

$$\tilde{W}_{i2} = \tilde{W}_{i1} - \theta_{i1} P_1 + \theta_{i1}(\tilde{V} + \tilde{\mu}) + \hat{X}_i(\tilde{L} + \tilde{e}), \tag{50}$$

and

$$\tilde{W}_{i1} = (\bar{\theta} - \theta_{i0}) P_0 + \theta_{i0} P_1, \tag{51}$$

and short-selling constraints (48), where $\delta > 0$ is the absolute risk-aversion parameter.

Both hedgers and non-hedgers have to trade through the market-maker, *i.e.*,

$$\theta_{mt} = N\bar{\theta} - (N_h \theta_{ht} + N_n \theta_{nt}), \quad t = 0, 1. \tag{52}$$

The market-maker's problem is then

$$\max_{P_0, P_1} E[-e^{-\delta \tilde{W}_{m2}}], \tag{53}$$

subject to the budget constraints

$$\tilde{W}_{m2} = \tilde{W}_{m1} - \theta_{m1} P_1 + \theta_{m1}(\tilde{V} + \tilde{\mu}), \tag{54}$$

and

$$\tilde{W}_{m1} = (\bar{\theta} - \theta_{m0}) P_0 + \theta_{m0} P_1. \tag{55}$$

Definition 2 *An equilibrium* $(\theta^*_{ht}, \theta^*_{nt}, P^*_t, t = 0, 1)$ *is such that*

[37] To simplify exposition, we use θ_i to denote the holdings after trading instead of the trading amount as used previously.

1. *Given* P_t^*, θ_{it}^* $(i \in \{h,n\})$ *solves investor i's Problem* (49) − (51) *for* $t = 0, 1$*;*

2. *Given* θ_{ht}^* *and* θ_{nt}^*, P_t^* *solves the market-maker's Problem (53)-(55) for* $t = 0, 1$ *subject to the market-clearing condition (52).*

In general, there are 16 possible cases, depending on when the constraints bind for whom (hedgers or non-hedgers). To simplify the exposition and save space, we report the equilibrium results only for the case in which there are no non-hedgers, i.e., $N_n = 0$, because in this case, there are only four possible cases in equilibrium.[38]

Define

$$C_0 := \frac{\sigma_\mu^2 + (N_h+2)^2\sigma_V^2}{(N_h+2)\sigma_{VL}}\bar{\theta} > D_0 := \frac{(N_h+2)\sigma_V^2}{\sigma_{VL}}\bar{\theta}, \tag{56}$$

$$C_1 := \frac{(N_h+2)\sigma_\mu^2(\sigma_\mu^2 + (N_h+2)^2\sigma_V^2)}{(N_h+2)^2\sigma_V^2\sigma_{\mu e} + \sigma_\mu^2((N_h+2)\sigma_{VL} + \sigma_{\mu e})}\bar{\theta}, \quad D_1 := \frac{(N_h+1)\sigma_\mu^2}{\sigma_{\mu e}}\bar{\theta}. \tag{57}$$

The following theorem provides the equilibrium prices and equilibrium security demand in closed form.[39]

Theorem 3 1. *If* $\hat{X}_h < \min\{C_0, C_1\}$, *then short-sale constraints do not bind for hedgers at either time 0 or time 1,*

(a) *the equilibrium prices at time 0 and 1 are*

$$P_0^* = \bar{V} - \frac{\delta(N_h+1)\left(\sigma_\mu^2\sigma_{\mu e} + (N_h+2)^2\sigma_V^2(\sigma_{VL} + \sigma_{\mu e})\right)\hat{X}_h}{(N_h+2)\left(\sigma_\mu^2 + (N_h+2)^2\sigma_V^2\right)} - \delta(\sigma_\mu^2 + \sigma_V^2)\bar{\theta}, \tag{58}$$

$$P_1^* = \tilde{V} + \delta\left(\frac{\sigma_\mu^2\sigma_{VL}}{\sigma_\mu^2 + (N_h+2)^2\sigma_V^2} - \frac{(N_h+1)\sigma_{\mu e}}{N_h+2}\right)\hat{X}_h - \delta\sigma_\mu^2\bar{\theta}, \tag{59}$$

(b) *and the equilibrium quantities demanded at time 0 and 1 are*

$$\theta_{h0}^* = \bar{\theta}\left(1 - \frac{\hat{X}_h}{C_0}\right), \quad \theta_{m0}^* = (N_h+1)\bar{\theta} - N_h\theta_{h0}^*, \tag{60}$$

[38] The presence of non-hedgers is important for the existence of different bid and ask prices, but not for the determination of equilibrium prices when bid-ask spread is restricted to be zero, as we consider here. Indeed, we also solved the equilibrium with short-sale constraints when $N_n \neq 0$. The qualitative results are the same and available from the authors.

[39] Because the second period problem is essentially the same as the one-period problem except for different initial endowments, and the first period problem is similar to the one-period problem except for different continuation utilities, the proof is straightforward and thus omitted.

$$\theta^*_{h1} = \bar{\theta}\left(1 - \frac{\hat{X}_h}{C_1}\right), \quad \theta^*_{m1} = (N_h+1)\bar{\theta} - N_h\theta^*_{h1}; \tag{61}$$

2. *If $C_1 \leq \hat{X}_h < D_0$, then short-sale constraints bind for hedgers only at time 1,*

   (a) *the equilibrium prices at time 0 and 1 are*

$$P^*_{0c1} = \bar{V} - \delta\left(\sigma_{\mu e} + \frac{N_h+1}{N_h+2}\sigma_{VL}\right)\hat{X}_h - \delta\sigma_V^2\bar{\theta}, \quad P^*_{1c1} = \tilde{V} - \delta\sigma_{\mu e}\hat{X}_h, \tag{62}$$

   (b) *and the equilibrium quantities demanded at time 0 and 1 are*

$$\theta^*_{h0c1} = \bar{\theta}\left(1 - \frac{\hat{X}_h}{D_0}\right), \quad \theta^*_{m0c1} = (N_h+1)\bar{\theta} - N_h\theta^*_{h0c1}, \quad \theta^*_{h1c1} = 0, \quad \theta_{m1c1} = (N_h+1)\bar{\theta}; \tag{63}$$

3. *If $C_0 \leq \hat{X}_h < D_1$, then short-sale constraints bind for hedgers only at time 0,*

   (a) *the equilibrium prices at time 0 and 1 are*

$$P^*_{0c2} = \bar{V} - \delta\sigma_{VL}\hat{X}_h - \frac{N_h+1}{N_h+2}\delta\sigma_{\mu e}\hat{X}_h - \frac{N_h+1}{N_h+2}\delta\sigma_\mu^2\bar{\theta}, \tag{64}$$

$$P^*_{1c2} = \tilde{V} - \frac{N_h+1}{N_h+2}\delta\sigma_{\mu e}\hat{X}_h - \frac{N_h+1}{N_h+2}\delta\sigma_\mu^2\bar{\theta}, \tag{65}$$

   (b) *and the equilibrium quantities demanded at time 0 and 1 are*

$$\theta^*_{h0c2} = 0, \quad \theta^*_{m0c2} = (N_h+1)\bar{\theta}, \tag{66}$$

$$\theta^*_{h1c2} = \frac{N_h+1}{N_h+2}\bar{\theta}\left(1 - \frac{\hat{X}_h}{D_1}\right), \quad \theta^*_{m1c2} = (N_h+1)\bar{\theta} - N_h\theta^*_{h1c2}; \tag{67}$$

4. *Otherwise, short-sale constraints bind for hedgers at both time 0 and 1,*

   (a) *the equilibrium prices at time 0 and 1 are*

$$P^*_{0c3} = \bar{V} - \delta\left(\sigma_{VL} + \sigma_{\mu e}\right)\hat{X}_h, \quad P^*_{1c3} = \tilde{V} - \delta\sigma_{\mu e}\hat{X}_h, \tag{68}$$

   (b) *and the equilibrium quantities demanded at time 0 and 1 are*

$$\theta^*_{h0c2} = \theta^*_{h1c2} = 0, \quad \theta^*_{m0c2} = \theta^*_{m1c2} = (N_h+1)\bar{\theta}. \tag{69}$$

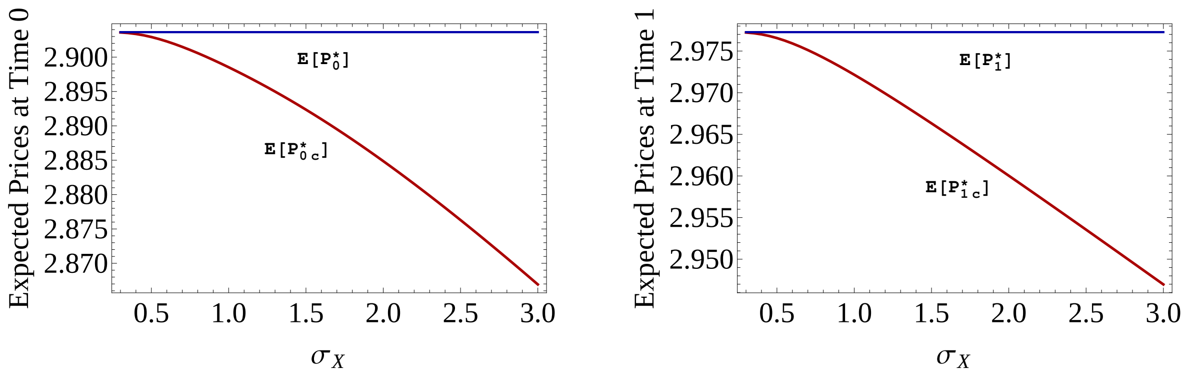


Figure 11: The expected prices at time 0 and 1 with and without short-sale constraints against σ_X. The parameter values are: $\delta = 1$, $\sigma_V = 0.9$, $\sigma_{VL} = 0.3$, $\sigma_\mu = 0.5, \sigma_{\mu e} = 0.4$, $\bar{V} = 3$, $\tilde{V} = 3$, $N_h = 10$, $\bar{\theta} = 1/(N_h + 1)$.

Theorem 3 implies that, as in the main model, short-sale constraints can lower the equilibrium trading price when a buyer has market power. For example, in Case 2, in which hedgers are constrained at time 1 only, one can show that the equilibrium prices at both time 0 and time 1 are lower than those without short-sale constraints respectively. To examine the average impact of short-sale constraints on equilibrium prices, we next plot in Figure 11 the expected equilibrium prices (across $\hat{X}_h$) at time 0 (left subfigure) and at time 1 (right subfigure) with and without short-sale constraints against the liquidity shock volatility σ_X. Figure 11 shows that the expected prices at time 0 and time 1 with short-sale constraints are lower than the prices without short-sale constraints, as in our one-period model. These results hold because the market power of the market-maker constitutes the main driving force, which is still present in a dynamic setting.

7. Conclusions

Regulatory short-sale constraints are often imposed when market conditions deteriorate and markets become much less competitive. In contrast, extant theories on how short-sale constraints affect asset prices and market liquidity exclusively focus on perfectly competitive markets, and cannot explain the robust empirical finding that impositions of regulatory short-sale bans cause significant increases in bid-ask spreads in many financial markets. In this paper, we demonstrate that the impact of short-sale constraints in an imperfectly competitive market in which market-makers have market power is qualitatively different from that in a perfectly competitive market. Our model predicts that short-sale constraints drive bid prices down and bid-ask spreads up. If, in addition, market-makers are risk-averse, then short-sale constraints also drive the ask price up. Furthermore, short-sale constraints increase liquidity-risk as measured by the volatility of bid-ask spreads. The main results are largely unaffected by the presence of information asymmetry, endogenization of information acquisition, reduced information revelation, or dynamic trading. Moreover, more public disclosure can further magnify the adverse impact of short-sale constraints on asset prices and market liquidity.

Our model provides some novel empirically-testable implications. For example, in markets in which market-makers have significant market power, short-sale constraints decrease average bid, but increase average spread and spread volatility; and the impact of short-sale constraints is greater in more transparent markets.

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Appendix A

We first state the main results for the extended model with asymmetric information. Let Δ denote the difference in the reservation prices of h and n investors, i.e.,

$$\Delta := P_h^R - P_n^R = (1-\rho_n)\left(\left(1+\frac{\sigma_v^2}{\rho_h\sigma_\eta^2}\right)\hat{S} - \frac{\sigma_v^2}{\sigma_\eta^2}\hat{S}_s + \delta\rho_h\sigma_v^2\bar{\theta}\right). \tag{A-1}$$

Let

$$\nu := \frac{\mathrm{Var}[\tilde{V}|\mathcal{I}_n]}{\mathrm{Var}[\tilde{V}|\mathcal{I}_h]} \geq 1$$

be the ratio of the security payoff conditional variance of non-hedgers to that of hedgers, and

$$\overline{N} := \nu N_h + N_n + 1 \geq N$$

be the information weighted total population. The following theorem provides the equilibrium bid and ask prices and equilibrium security demand in closed form.

Theorem 4 *1. If* $-\frac{2\left(\overline{N}+1\right)\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta})}{N_n+2} < \Delta < \frac{2\left(\overline{N}+1\right)\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta})}{\nu N_h}$, *then short-sale constraints do not bind for any investors,*

(a) the equilibrium bid and ask prices are

$$A^* = P_n^R + \frac{\nu N_h}{2\left(\overline{N}+1\right)}\Delta + \frac{\Delta^+}{2}, \tag{A-2}$$

$$B^* = P_n^R + \frac{\nu N_h}{2\left(\overline{N}+1\right)}\Delta - \frac{\Delta^-}{2}, \tag{A-3}$$

the bid-ask spread is

$$A^* - B^* = \frac{|\Delta|}{2} = \frac{(1-\rho_n)\left|\left(1+\frac{\sigma_v^2}{\rho_h\sigma_\eta^2}\right)\hat{S} - \frac{\sigma_v^2}{\sigma_\eta^2}\hat{S}_s + \delta\rho_h\sigma_v^2\bar{\theta}\right|}{2}, \tag{A-4}$$

(b) the equilibrium quantities demanded are

$$\theta_h^* = \frac{N_n+2}{2\left(\overline{N}+1\right)\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_h]}\Delta, \quad \theta_n^* = -\frac{\nu N_h}{2\left(\overline{N}+1\right)\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_n]}\Delta, \quad \theta_m^* = 2\theta_n^*, \tag{A-5}$$

and the equilibrium quote depths are

$$\alpha^* = N_h(\theta_h^*)^+ + N_n(\theta_n^*)^+, \tag{A-6}$$

$$\beta^* = N_h(\theta_h^*)^- + N_n(\theta_n^*)^-; \tag{A-7}$$

2. *If* $\Delta \leq -\frac{2\left(\overline{N}+1\right)\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta})}{N_n+2}$, *then short-sale constraints bind for hedgers,*

   (a) *the equilibrium bid and ask prices are*

$$A_{c1}^* = P_n^R - \frac{\delta\nu N_h \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta})}{N_n+2}, \tag{A-8}$$

$$B_{c1}^* = P_h^R + \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta}), \tag{A-9}$$

the bid-ask spread is

$$A_{c1}^* - B_{c1}^* = -\Delta - \frac{\bar{N}+1}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta}), \tag{A-10}$$

   (b) *the equilibrium quantities demanded are*

$$\theta_{hc1}^* = -(\kappa_h+\bar{\theta}), \quad \theta_{nc1}^* = \frac{N_h(\kappa_h+\bar{\theta})}{N_n+2}, \ \theta_{mc1}^* = \frac{2N_h(\kappa_h+\bar{\theta})}{N_n+2}, \tag{A-11}$$

and the equilibrium quote depths are

$$\alpha_{c1}^* = \frac{N_h N_n(\kappa_h+\bar{\theta})}{N_n+2}, \ \beta_{c1}^* = N_h(\kappa_h+\bar{\theta}); \tag{A-12}$$

3. *If* $\Delta \geq \frac{2\left(\overline{N}+1\right)\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta})}{\nu N_h}$, *then short-sale constraints bind for non-hedgers,*

   (a) *the equilibrium bid and ask prices are*

$$A_{c2}^* = P_h^R - \frac{\Delta + \delta N_n \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta})}{\nu N_h+2}, \tag{A-13}$$

$$B_{c2}^* = P_n^R + \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta}), \tag{A-14}$$

the bid-ask spread is

$$A^*_{c2} - B^*_{c2} = \frac{\nu N_h + 1}{\nu N_h + 2}\Delta - \frac{\bar{N} + 1}{\nu N_h + 2}\delta \text{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta}), \tag{A-15}$$

(b) the equilibrium quantities demanded are

$$\theta^*_{hc2} = \frac{\Delta + \delta N_n \text{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 2)\delta \text{Var}[\tilde{V}|\mathcal{I}_h]}, \quad \theta^*_{nc2} = -(\kappa_n + \bar{\theta}), \tag{A-16}$$

$$\theta^*_{mc2} = \frac{-\nu N_h \Delta + 2\delta N_n \text{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 2)\delta \text{Var}[\tilde{V}|\mathcal{I}_n]}, \tag{A-17}$$

and the equilibrium quote depths are

$$\alpha^*_{c2} = \frac{N_h \Delta + \delta N_h N_n \text{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 2)\delta \text{Var}[\tilde{V}|\mathcal{I}_h]}, \ \beta^*_{c2} = N_n(\kappa_n + \bar{\theta}). \tag{A-18}$$

Proof of Theorems 1 and 4:

We only prove the generalized model with information asymmetry, because it nests the main model with symmetric information by setting $\sigma_\varepsilon = \infty$. We consider the case when $\Delta < 0$, and the other case is similar. In this case, we conjecture that h investors sell and n investors buy. First, suppose no investors are constrained. Given bid price B and ask price A, the optimal demand of h and n are respectively:

$$\theta^*_h = \frac{P^R_h - B}{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]} \quad \textit{and} \quad \theta^*_n = \frac{P^R_n - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_n]}. \tag{A-19}$$

Substituting (A-19) into the market-clearing condition (4), we obtain that the market-clearing ask and bid depths are respectively:

$$\beta = -N_h \theta^*_h = N_h \frac{B - P^R_h}{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]}, \quad \alpha \quad = \quad N_n \theta^*_n = N_n \frac{P^R_n - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_n]}. \tag{A-20}$$

Because of the CARA utility and the normal distribution of the date 1 wealth, the market-maker's problem is equivalent to:

$$\max_{A,B} \alpha A - \beta B + (\bar{\theta} + \beta - \alpha)E[\tilde{V}|\mathcal{I}_m] - \frac{1}{2}\delta \text{Var}[\tilde{V}|\mathcal{I}_m](\bar{\theta} + \beta - \alpha)^2, \tag{A-21}$$

subject to (A-20). The F.O.C with respect to B (noting that β is a function of B) gives us:

$$-\beta - B\frac{N_h}{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]} + E[\tilde{V}|\mathcal{I}_m]\frac{N_h}{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]} - \delta \text{Var}[\tilde{V}|\mathcal{I}_m](\bar{\theta} + \beta - \alpha)\frac{N_h}{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]} = 0,$$

which can be reduced to

$$(\nu N_h + 2)\beta - \nu N_h \alpha = -\frac{N_h \Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]}, \tag{A-22}$$

by using (36) and expressing B in terms of β using (A-20).

Similarly using the F.O.C with respect to A, we obtain

$$\alpha + A\left(-\frac{N_n}{\delta \text{Var}[\tilde{V}|\mathcal{I}_n]}\right) - E[\tilde{V}|\mathcal{I}_m]\left(-\frac{N_n}{\delta \text{Var}[\tilde{V}|\mathcal{I}_n]}\right)$$
$$+\delta \text{Var}[\tilde{V}|\mathcal{I}_m](\bar{\theta} + \beta - \alpha)\left(-\frac{N_n}{\delta \text{Var}[\tilde{V}|\mathcal{I}_n]}\right) = 0, \tag{A-23}$$

which can be reduced to

$$(N_n + 2)\alpha - N_n \beta = 0, \tag{A-24}$$

by using (36), expressing A in terms of α using (A-20), and noting that $\mathcal{I}_m = \mathcal{I}_n$.

Solving (A-24) and (A-22), we can obtain the equilibrium ask depth and bid depth α^* and β^* as in (A-6) and (A-7). Substituting α^* and β^* into (A-20), we can obtain the equilibrium ask and bid prices A^* and B^* as in (A-2) and (A-3). In addition, by the market-clearing condition, we have $\theta_n^* = \alpha^*/N_n$, $\theta_h^* = -\beta^*/N_h$, $\theta_m^* = \beta^* - \alpha^*$, which can be simplified into Equation (A-5).

The short-sale constraints bind for hedgers if and only if $\theta_h^* \leq -(\kappa_h + \bar{\theta})$, equivalently, if and only if $\Delta \leq -\frac{2(\overline{N}+1)\delta \text{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta})}{N_n + 2}$. When short-sale constraints bind for hedgers, we have $\theta_{hc1}^* = -(\kappa_h + \bar{\theta})$ and $\beta_{c1}^* = N_h(\kappa_h + \bar{\theta})$. Because the first order condition (A-24) with respect to α remains the same, we have:

$$\alpha_{c1}^* = \frac{N_h N_n}{N_n + 2}(\kappa_h + \bar{\theta}).$$

Then from (A-20), we get the equilibrium bid price B_{c1}^* and ask price A_{c1}^* when short-sale constraints bind for hedgers. Other quantities can then be derived. Similarly, we can prove Theorem 4 for the other case in which h investors buy and n investors sell. In addition, there is no equilibrium where the composite signal $\hat{S}$ is not fully revealed. This is because if the composite signal $\hat{S}$ is low enough such that the equilibrium price without short-sale constraints would make the short-sale constraints strictly bind, then a bid price that is lower than the threshold price at which the short-sale constraints start to bind would make the short-sale constraints non-binding, and the market-maker would be better off by increasing the bid price so that she can buy more from the sellers. *Q.E.D.*

Proof of Proposition 1: We prove this proposition for the case in which h investors sell, the proof of the other case is very similar and we thus skip it here. Conditional on the constraint binding for hedgers, it is clear from Theorem 4 that A^*_{c1} decreases in κ_h, B^*_{c1} increases in κ_h and $(A^*_{c1} - B^*_{c1})$ decreases in κ_h. We next show that compared to the case without short-sale constraints, the bid price is lower and the ask price is higher with the constraints. By Theorem 4, we have

$$B^*_{c1} - B^* = \frac{N_n + 2}{2(\bar{N}+1)}\Delta + \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta}), \tag{A-25}$$

and

$$A^*_{c1} - A^* = -\frac{\nu N_h}{2(\bar{N}+1)}\Delta - \frac{N_h}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_h + \bar{\theta}). \tag{A-26}$$

The condition $\Delta < -\frac{2(\bar{N}+1)}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta})$ implies that $B^*_{c1} \leq B^*$ and $A^*_{c1} \geq A^*$, which leads to $A^*_{c1} - B^*_{c1} \geq A^* - B^*$. Similarly, the results on depths and trading volume can be demonstrated. *Q.E.D.*

The following lemma is used to prove Proposition 3.

Lemma 1 *Let $f(x) := |x|$ and*

$$g(x) := \begin{cases} c_1(k_1 x - h_1) & x > \frac{h_1}{k_1} \\ c_2(-k_2 x - h_2) & x < -\frac{h_2}{k_2}, \\ 0 & \textit{otherwise}, \end{cases}$$

where k_1, k_2, h_1, h_2, c_1 and c_2 are positive constants and x is randomly distributed in $(-\infty, +\infty)$ with probability density function $p(x)$ which is an even function. Then we have $Cov(f(x), g(x)) > 0$.

Proof :

$$\begin{aligned} &Cov(f(x), g(x)) = E(f(x)g(x)) - E(f(x))E(g(x)) \\ &= \int_{-\infty}^{+\infty} f(x)g(x)p(x)dx - \int_{-\infty}^{+\infty} f(x)p(x)dx \int_{-\infty}^{+\infty} g(x)p(x)dx \\ &= \int_{-\infty}^{+\infty} p(y)dy \int_{-\infty}^{+\infty} f(x)g(x)p(x)dx - \int_{-\infty}^{+\infty} f(y)p(y)dy \int_{-\infty}^{+\infty} g(x)p(x)dx \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(f(x)g(x) - f(y)g(x)\right) p(x)p(y)dxdy \\ &= \frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(f(x) - f(y)\right)\left(g(x) - g(y)\right) p(x)p(y)dxdy. \end{aligned} \tag{A-27}$$

Since $p(-x) = p(x)$ and $p(-y) = p(y)$, we have

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (f(-x) - f(-y)) (g(-x) - g(-y)) p(x)p(y)dxdy$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (f(x) - f(y)) (g(x) - g(y)) p(x)p(y)dxdy. \tag{A-28}$$

From (A-27) and (A-28), we have $Cov(f(x), g(x)) =$

$$\frac{1}{4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (f(x) - f(y)) (g(x) + g(-x) - g(y) - g(-y)) p(x)p(y)dxdy. \tag{A-29}$$

(1) If x and y have the same sign, the term inside of the integral can be written as

$$(f(x) - f(y)) (g(x) - g(y)) + (f(-x) - f(-y)) (g(-x) - g(-y)),$$

which is ≥ 0.

(2) If $x < 0$ and $y > 0$, the term inside of the integral can be written as

$$(f(-x) - f(y)) (g(-x) - g(y)) + (f(x) - f(-y)) (g(x) - g(-y)),$$

which is ≥ 0.

(3) If $x > 0$ and $y < 0$, the term inside of the integral can be written as

$$(f(x) - f(-y)) (g(x) - g(-y)) + (f(-x) - f(y)) (g(-x) - g(y)),$$

which is ≥ 0. In addition, at least for some x and y, the term inside of the integral is non-zero. Therefore, $Cov(f(x), g(x)) > 0$. *Q.E.D.*

Proof of Proposition 2: Because the market-maker is risk-neutral, she chooses A and B to maximize $A\,\alpha(A) - B\,\beta(B) + (\bar{\theta} + \beta(B) - \alpha(A))\bar{V}$ and thus the choices of B and A are independent. Therefore, short-sale constraints have no impact on ask or ask depth. In addition, short-sale constraints still reduce bid and bid depth by the same proof as above. *Q.E.D.*

Proof of Proposition 3: The spread with short-sale constraints $A_c^* - B_c^*$ can be written as $f(\Delta) + g(\Delta)$, where

$$f(\Delta) = A^* - B^* = \frac{|\Delta|}{2}$$

and

$$g(\Delta) = \begin{cases} \frac{\nu N_h}{2(\nu N_h+2)}\Delta - \frac{\bar{N}+1}{\nu N_h+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta}) & \Delta \geq \frac{2(\bar{N}+1)}{\nu N_h}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta}), \\ -\frac{1}{2}\Delta - \frac{\bar{N}+1}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta}) & \Delta \leq -\frac{2(\bar{N}+1)}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta}), \\ 0 & \textit{otherwise}. \end{cases}$$

By Lemma 1, $f(\Delta)$ and $g(\Delta)$ are positively correlated. Then it follows that
$\mathrm{Var}(A_c^* - B_c^*) > \mathrm{Var}(A^* - B^*)$. *Q.E.D.*

Proof of Theorem 2: The proof is straightforward. Here we only outline the proof. Define

$$\nu_1 = \frac{\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}{\mathrm{Var}[\tilde{V}_h|\mathcal{I}_h]},\ \nu_2 = \frac{\mathrm{Var}[\tilde{V}_m|\mathcal{I}_m]}{\mathrm{Var}[\tilde{V}_h|\mathcal{I}_h]},$$

$$\varphi_m = \frac{\delta_h\delta_n\nu_1}{\delta_m\delta_n\nu_1\nu_2N_h + 2\delta_h\delta_n\nu_1 + \delta_m\delta_h\nu_2N_n}, \tag{A-30}$$

$$\varphi_n = N_n\left(\frac{\delta_m\nu_2}{\delta_n\nu_1} + \frac{\delta_h}{N_n\delta_h + N_h\nu_1\delta_n}\right)\varphi_m, \tag{A-31}$$

$$\lambda_m = \frac{\delta_n\nu_1}{N_n\delta_m\nu_2 + 2\delta_n\nu_1}, \lambda_h = \frac{N_n\delta_m\nu_2 + \delta_n\nu_1}{N_n\delta_m\nu_2 + 2\delta_n\nu_1}\frac{\delta_n\nu_1N_h}{\delta_hN_n}, \tag{A-32}$$

$$\gamma_m = \frac{\delta_h}{N_h\delta_m\nu_2 + 2\delta_h}, \gamma_n = \frac{N_h\delta_m\nu_2 + \delta_h}{N_h\delta_m\nu_2 + 2\delta_h}\frac{\delta_hN_n}{\delta_n\nu_1N_h}. \tag{A-33}$$

First, assuming that there are no short-sale constraints, then the market-maker's problem is equivalent to

$$\begin{aligned} \max_P \quad & N_h\tfrac{P_h^R-P}{\delta_h\mathrm{Var}[\tilde{V}_h]}P + N_n\tfrac{P_n^R-P}{\delta_n\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}P + \left(\bar{\theta}_m + N_h\tfrac{P-P_h^R}{\delta_h\mathrm{Var}[\tilde{V}_h|\mathcal{I}_h]} + N_n\tfrac{P-P_n^R}{\delta_n\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}\right) \\ & \times(P_m^R + \delta_m\mathrm{Var}[\tilde{V}_m|\mathcal{I}_m]\bar{\theta}_m) - \tfrac{1}{2}\delta_m\mathrm{Var}[\tilde{V}_m|\mathcal{I}_m]\left(\bar{\theta}_m + N_h\tfrac{P-P_h^R}{\delta_h\mathrm{Var}[\tilde{V}_h|\mathcal{I}_h]} + N_n\tfrac{P-P_n^R}{\delta_n\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}\right)^2. \end{aligned}$$

The first order condition then yields P^*. Conditional on hedgers always selling $\kappa_h + \bar{\theta}_h$, the market-maker's problem is equivalent to

$$\begin{aligned} \max_P \quad & -N_h(\kappa_h+\bar{\theta}_h)P + N_n\tfrac{P_n^R-P}{\delta_n\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}P + \left(\bar{\theta}_m + N_h(\kappa_h+\bar{\theta}_h) + N_n\tfrac{P-P_n^R}{\delta_n\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}\right) \\ & \times(P_m^R + \delta_m\mathrm{Var}[\tilde{V}_m|\mathcal{I}_m]\bar{\theta}_m) - \tfrac{1}{2}\delta_m\mathrm{Var}[\tilde{V}_m|\mathcal{I}_m]\left(\bar{\theta}_m + N_h(\kappa_h+\bar{\theta}_h) + N_n\tfrac{P-P_n^R}{\delta_n\mathrm{Var}[\tilde{V}_n|\mathcal{I}_n]}\right)^2. \end{aligned}$$

The first order condition then yields P_c^h. Similarly, conditional on non-hedgers always selling $\kappa_n + \bar{\theta}_n$, we can derive the equilibrium price P_c^n. Then the comparison of the maximum expected utility with the constraint that $P \leq \bar{P}_h$ and the maximum expected utility with the constraint that $\bar{P}_h \leq P \leq \bar{P}_n$, while noting that the maximum expected utility with the constraint that $P \geq \bar{P}_n$ is equal to the expected utility at $P = \bar{P}_n$, yields the equilibrium prices under different conditions. *Q.E.D.*

Robustness with reduced information revelation when hedgers are constrained

We assume that the informed (hedgers) are not endowed with any shares of the stock, the market-maker is endowed with $\bar{\theta}_m$ shares of the stock, and each uninformed trader (nonhedger) is endowed with $\bar{\theta}_n$ shares of the stock. For tractability, we study the case in which the market-maker has to post $A = B = P$. To simplify computations, we also assume that there is no public signal $\hat{S}_s$, *i.e.*, $\sigma_\eta = \infty$.

It can be demonstrated that hedgers are constrained by short-sale constraints and thus are not trading when

$$\hat{S} \leq \underline{S} := -\frac{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]\nu \left((N_h\nu + N_n)(\bar{\theta}_m + N_n\bar{\theta}_n) + N_n\bar{\theta}_n\right)}{(1-\rho_n)(N_h\nu(N_n+1) + N_n(N_n+2))}.$$

When the informed are constrained by short-sale constraints, in equilibrium, there are two cases: 1) if $\bar{\theta}_m > \bar{\theta}_n$, the uninformed buy from the market-maker; and 2) if $\bar{\theta}_m < \bar{\theta}_n$, then (i) the uninformed sell, and they are not constrained by short-sale constraints when $\underline{S}_n < \hat{S} < \underline{S}$, (ii) the uninformed buy when $\hat{S} < \underline{S}_n$, where

$$\underline{S}_n = \frac{\delta \text{Var}[\tilde{V}|\mathcal{I}_h]\left((N_h\nu + N_n)\bar{\theta}_m - (N_n + N_h\nu(N_h\nu + N_n + 2))\bar{\theta}_n\right)}{(1-\rho_n)N_h(N_h\nu + N_n + 1)}.$$

We present the details of case (1), *i.e.*, $\bar{\theta}_m > \bar{\theta}_n$, the uninformed buy from the market-maker. Case (2) can be solved similarly.

When the informed are constrained by short-sale constraints and they are not endowed with any shares of a risky asset, informed traders are not trading. The market-maker and the uninformed only know that $\hat{S} \leq \underline{S}$, and they cannot observe $\hat{S}$. The uninformed's problem becomes

$$\max_{\theta_n} E[-e^{-\delta\left(-\theta_n P + (\bar{\theta}_n + \theta_n)(\tilde{v} + \tilde{u})\right)}|\hat{S} \leq \underline{S}]. \tag{A-34}$$

It can be shown that (A-34) is equivalent to

$$\min_{\theta_n} e^{\delta\theta_n P + \frac{1}{2}\delta^2(\bar{\theta}_n + \theta_n)^2(\sigma_u^2 + \sigma_v^2) - \delta(\bar{\theta}_n + \theta_n)\bar{V}} \mathbf{N}\left(\frac{\underline{S} + \delta(\bar{\theta}_n + \theta_n)\rho_h\sigma_v^2}{\sqrt{\rho_h\sigma_v^2 + \omega^2\sigma_X^2}}\right) / \mathbf{N}\left(\frac{\underline{S}}{\sqrt{\rho_h\sigma_v^2 + \omega^2\sigma_X^2}}\right). \tag{A-35}$$

Taking the first-order condition with respect to θ_n in equation (A-35) yields

$$(P+\delta(\bar{\theta}_n+\theta_n)(\sigma_u^2+\sigma_v^2)-\bar{V})\mathbf{N}\left(\frac{\underline{S}+\delta(\bar{\theta}_n+\theta_n)\rho_h\sigma_v^2}{\sqrt{\rho_h\sigma_v^2+\omega^2\sigma_X^2}}\right)+\frac{1}{\sqrt{2\pi}}\frac{\rho_h\sigma_v^2}{\sqrt{\rho_h\sigma_v^2+\omega^2\sigma_X^2}}e^{-\frac{(\underline{S}+\delta(\bar{\theta}_n+\theta_n)\rho_h\sigma_v^2)^2}{2(\rho_h\sigma_v^2+\omega^2\sigma_X^2)}}=0. \tag{A-36}$$

The market-maker's problem becomes

$$\max_P \; E[-e^{-\delta\left(\alpha P+(\bar{\theta}_m-\alpha)(\tilde{v}+\tilde{u})\right)}|\hat{S}\leq\underline{S}]. \tag{A-37}$$

It can be shown that (A-37) is equivalent to

$$\min_P \; e^{-\delta P\alpha+\frac{1}{2}\delta^2(\bar{\theta}_m-\alpha)^2(\sigma_u^2+\sigma_v^2)-\delta(\bar{\theta}_m-\alpha)\bar{V}}\mathbf{N}\left(\frac{\underline{S}+\delta(\bar{\theta}_m-\alpha)\rho_h\sigma_v^2}{\sqrt{\rho_h\sigma_v^2+\omega^2\sigma_X^2}}\right)/\mathbf{N}\left(\frac{\underline{S}}{\sqrt{\rho_h\sigma_v^2+\omega^2\sigma_X^2}}\right). \tag{A-38}$$

Taking the first order condition with respect to P in equation (A-38) yields

$$\left(\alpha+\frac{\partial\alpha}{\partial P}(P+\delta(\bar{\theta}_m-\alpha)(\sigma_u^2+\sigma_v^2)-\bar{V})\right)\mathbf{N}\left(\frac{\underline{S}+\delta(\bar{\theta}_m-\alpha)\rho_h\sigma_v^2}{\sqrt{\rho_h\sigma_v^2+\omega^2\sigma_X^2}}\right)$$
$$+\frac{1}{\sqrt{2\pi}}\frac{\rho_h\sigma_v^2}{\sqrt{\rho_h\sigma_v^2+\omega^2\sigma_X^2}}\frac{\partial\alpha}{\partial P}e^{-\frac{(\underline{S}+\delta(\bar{\theta}_m-\alpha)\rho_h\sigma_v^2)^2}{2(\rho_h\sigma_v^2+\omega^2\sigma_X^2)}}=0. \tag{A-39}$$

From (A-36), we can express P and $\frac{\partial\theta_n}{\partial P}$ as functions of θ_n. Substituting $\alpha=N_n\theta_n$ and $\frac{\partial\alpha}{\partial P}=N_n\frac{\partial\theta_n}{\partial P}$ into (A-39) yields a function of θ_n which can be solved numerically and then we obtain the equilibrium price. *Q.E.D.*

Appendix B

Equilibrium with a price-taking market-maker

To show the impact of the market-maker's market power on equilibrium results, we next consider two cases in which the market-maker has a weaker market power.

First, to be self-contained, we consider the case in which the market-maker is a price-taker in both the ask market and the bid market and thus in equilibrium the bid price must be equal to the ask price, as studied in the extant literature. Let P denote the stock price. Given P, the optimal demand schedule for a type i investor for $i \in \{h, n\}$ is

$$\theta_i^*(P) = \max\left[-(\kappa_i + \bar{\theta}), -\frac{P - P_i^R}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_i]}\right]. \quad \text{(B-1)}$$

Solving for the equilibrium, we have[40]

Theorem 5 *1. If* $-\frac{\bar{N}\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta})}{N_n+1} < \Delta < \frac{\bar{N}\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta})}{\nu N_h}$*, then no one is constrained, and the equilibrium price is*

$$P^* = \frac{\nu N_h}{\bar{N}} P_h^R + \frac{N_n}{\bar{N}} P_n^R + \frac{1}{\bar{N}} P_m^R, \quad \text{(B-2)}$$

and the investors' optimal stock demand is given by

$$\theta_h^* = \frac{N_n + 1}{\bar{N}} \frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h]}, \quad \theta_n^* = \theta_m^* = -\frac{\nu N_h}{\bar{N}} \frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n]}; \quad \text{(B-3)}$$

2. If $\Delta \geq \frac{\bar{N}\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n+\bar{\theta})}{\nu N_h}$*, then short-sale constraints bind for non-hedgers, and the equilibrium price is*

$$P_{c1}^* = P_h^R - \frac{\Delta + N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + 1}, \quad \text{(B-4)}$$

and the investors' optimal stock demand is given by

$$\theta_{hc1}^* = \frac{\Delta + N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 1)\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h]}, \quad \theta_{nc1}^* = -(\kappa_n + \bar{\theta}), \quad \text{(B-5)}$$

$$\theta_{mc1}^* = \frac{-\nu N_h \Delta + N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 1)\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n]}; \quad \text{(B-6)}$$

3. If $\Delta \leq -\frac{\bar{N}\delta\mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h+\bar{\theta})}{N_n+1}$*, then short-sale constraints bind for hedgers, and the equilibrium price is*

$$P_{c2}^* = P_n^R - \frac{\nu N_h \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta})}{N_n + 1}, \quad \text{(B-7)}$$

and the investors' optimal stock demand is given by

$$\theta_{hc2}^* = -(\kappa_h + \bar{\theta}), \quad \theta_{nc2}^* = \theta_{mc2}^* = \frac{N_h(\kappa_h + \bar{\theta})}{N_n + 1}. \quad \text{(B-8)}$$

As shown by Theorem 5, the equilibrium price is a weighted average of the reservation prices of the

[40] The proofs of Theorems 5 and 6 are similar to those of Theorems 1 and 2, and are thus omitted.

investors in the economy. In addition, it is easy to show when short-sale constraints bind, the equilibrium selling price goes up (i.e., $P^*_{c1} > P^*$ and $P^*_{c2} > P^*$) and the trading volume decreases, as in the extant literature.

Theorem 5 assumes that the market-maker is a price-taker in both "bid" and "ask" markets. Next, to isolate the impact of the market power on how short-sale constraints affect bid price, we assume that the market-maker is a monopolist in the "ask" market, as in our main model, but is a price-taker in the "bid" market. Under this assumption, we have

Theorem 6 *1. If $0 < \Delta < \frac{\nu N_h + 2(N_n+1)}{\nu N_h}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})$, then no one is constrained, and the equilibrium prices are*

$$A_1^* = P_n^R + \frac{\bar{N}}{2(N_n+1)+\nu N_h}\Delta, \quad B_1^* = P_n^R + \frac{\nu N_h}{2(N_n+1)+\nu N_h}\Delta, \tag{B-9}$$

the bid-ask spread is

$$A_1^* - B_1^* = \frac{N_n+1}{2(N_n+1)+\nu N_h}\Delta,$$

the equilibrium depths are

$$\alpha_1^* = \frac{\nu N_h(N_n+1)}{2(N_n+1)+\nu N_h}\frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n]}, \quad \beta_1^* = \frac{N_n}{N_n+1}\alpha_1^*,$$

and the investors' optimal stock demand is given by

$$\theta_{h1}^* = \frac{\alpha_1^*}{N_h}, \quad \theta_{n1}^* = -\frac{\beta_1^*}{N_n}, \quad \theta_{m1}^* = -\frac{\alpha_1^*}{N_n+1}; \tag{B-10}$$

2. If $-\frac{2(\nu N_h+1)+N_n}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta}) < \Delta < 0$, then no one is constrained, and the equilibrium prices are

$$A_2^* = P_n^R + \frac{\nu N_h}{2\nu N_h + N_n + 2}\Delta, \quad B_2^* = P_n^R + \frac{2\nu N_h}{2\nu N_h + N_n + 2}\Delta \tag{B-11}$$

the bid-ask spread is

$$A_2^* - B_2^* = -\frac{\nu N_h}{2\nu N_h + N_n + 2}\Delta,$$

the equilibrium depths are

$$\alpha_2^* = -\frac{\nu N_h N_n}{N_n + 2 + 2\nu N_h}\frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n]}, \quad \beta_2^* = \frac{N_n+2}{N_n}\alpha_2^*,$$

and the investors' optimal stock demand is given by

$$\theta_{h2}^* = -\frac{\beta_2^*}{N_h}, \quad \theta_{n2}^* = \frac{\alpha_2^*}{N_n}, \quad \theta_{m2}^* = \frac{2}{N_n+2}\beta_2^*; \tag{B-12}$$

3. *If* $\Delta \geq \frac{\nu N_h + 2(N_n+1)}{\nu N_h}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})$*, then short-sale constraints bind for non-hedgers, and the equilibrium prices are*

$$A_{c1}^* = P_n^R + \frac{(\nu N_h + 1)\Delta - N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + 2}, \tag{B-13}$$

$$B_{c1}^* = P_n^R + \frac{\nu N_h \Delta - 2N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + 2}, \tag{B-14}$$

the bid-ask spread is

$$A_{c1}^* - B_{c1}^* = \frac{\Delta + N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + 2},$$

the equilibrium depths are

$$\alpha_{c1}^* = \frac{\nu N_h \Delta + \nu N_h N_n \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 2)\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n]}, \quad \beta_{c1}^* = N_n(\kappa_n + \bar{\theta}),$$

and the investors' optimal stock demand is given by

$$\theta_{hc1}^* = \frac{\alpha_{c1}^*}{N_h}, \quad \theta_{nc1}^* = -(\kappa_n + \bar{\theta}), \quad \theta_{mc1}^* = N_n(\kappa_n + \bar{\theta}) - \alpha_{c1}^*; \tag{B-15}$$

4. *If* $\Delta < -\frac{2(\nu N_h + 1) + N_n}{N_n + 2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_n](\kappa_h + \bar{\theta})$*, then short-sale constraints bind for hedgers, and the equilibrium prices are*

$$A_{c2}^* = P_n^R - \frac{\nu N_h}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta}), \quad B_{c2}^* = P_n^R - \frac{2\nu N_h}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta}), \tag{B-16}$$

the bid-ask spread is

$$A_{c2}^* - B_{c2}^* = \frac{\nu N_h}{N_n+2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta}),$$

the equilibrium depths are

$$\alpha_{c2}^* = \frac{N_n}{N_n+2}N_h(\kappa_h + \bar{\theta}), \quad \beta_{c2}^* = N_h(\kappa_h + \bar{\theta}),$$

and the investors' optimal stock demand is given by

$$\theta^*_{hc2} = -(\kappa_h + \bar{\theta}), \quad \theta^*_{nc2} = \frac{1}{N_n + 2} N_h(\kappa_h + \bar{\theta}), \quad \theta^*_{mc2} = \frac{2}{N_n + 2} N_h(\kappa_h + \bar{\theta}). \tag{B-17}$$

Given the results stated in Theorem 6, it is easy to show that as in our main model, short-sale constraints increase the equilibrium ask price and decrease bid/ask depths (and thus trading volume). In contrast to our main model, however, Theorem 6 suggests that short-sale constraints *increase* the equilibrium bid price (i.e., $B^*_{c1} > B^*_1$ and $B^*_{c2} > B^*_2$). Because the absence of market power of the market-maker in the bid market is the only difference from the main model, this shows that the key driving force behind the result that short-sale constraints decrease equilibrium bid prices is the market power of the market-maker.