Goal Programming

By Bruce Golden

Linear Programming Can Only Handle a Single Objective

- Is the heading True or False?
- Consider the toy problem below
  - A company produces Product 1, Product 2, and Product 3
  - Let x_i = the amount produced of Product i
  - Initial (optimistic) constraints
    1. $40x_1 + 30x_2 + 20x_3 \leq 100$ employees
    2. $2x_1 + 4x_2 + 3x_3 = 10$ tons of raw material
    3. $5x_1 + 8x_2 + 4x_3 \geq 30$ million dollars in profit
    4. $x_1, x_2, x_3 \geq 0$

Goal Programming

- Solving the toy problem
  - Given conflicting constraints, it is not possible that 1, 2, and 3 can all be satisfied
  - Therefore, management decides to penalize violations as follows
    1. 5 per unit over
    2. 8 per unit less, 12 per unit over
    3. 15 per unit less
- The following goal program emerges

Goal Programming Can Handle Multiple Objectives

$$\begin{array}{l} \text{Minimize } z = 5D_1^- + (12D_2^- + 8D_2^+) + 15D_3^+ \\ \text{subject to} \\ \begin{array}{llllllll} 40x_1 + & 30x_2 + & 20x_3 + & D_1^+ & - & D_1^- & = & 100 \\ 2x_1 & + \ 4x_2 & + \ 3x_3 & + \ D_2^+ & - & D_2^- & = & 10 \\ 5x_1 & + \ 8x_2 & + \ 4x_3 & + \ D_3^+ & - & D_3^- & = & 30 \end{array} \\ \text{all decision variables} \geq 0 \end{array}$$

- Answer: $x_2 = \frac{10}{3}$, $D_2^- = \frac{10}{3}$, $D_3^+ = \frac{10}{3}$, other decision variables $= 0$, $z = 90$
- This is a very simple goal program
- Goal programming can handle multiple objectives
- Goal programming has been used as an alternative to conventional (statistical) discriminant analysis (Freed & Glover, 1981)

An Alternative Goal Programming Model

$$
\begin{array}{lllllllll}
\text{Minimize } z = \gamma & & & & & & & & \\
\text{subject to} & & & & & & & & \\
 & & & 5D_1^- & & & & \leq & \gamma \\
 & & & 12D_2^- & + & 8D_2^+ & & \leq & \gamma \\
 & & & 15D_3^+ & & & & \leq & \gamma \\
40x_1 + & 30x_2 + & 20x_3 + & D_1^+ & - & D_1^- & & = & 100 \\
2x_1 + & 4x_2 + & 3x_3 + & D_2^+ & - & D_2^- & & = & 10 \\
5x_1 + & 8x_2 + & 4x_3 + & D_3^+ & - & D_3^- & & = & 30 \\
\text{all decision variables} \geq 0 & & & & & & & &
\end{array}
$$

- Answer: $x_1 = 0.29, x_2 = 3.22, D_1^- = 8.32, D_2^- = 3.47, D_3^+ = 2.77,$ other decision variables $= 0$, $z = 41.6$
- The idea of minimizing the maximum weighted deviation was due to (Flavell, 1976)