

Some Recent Developments in the Analytic Hierarchy Process

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AHP and Related Software

- Expert Choice (Forman)

EC Resource Aligner combines optimization with AHP to select the optimal combination of alternatives or projects subject to a budgetary constraint

- Criterium DecisionPlus (Hearne Scientific Software)

- HIPRE 3+ (Systems Analysis Laboratory, Helsinki)

- Web-HIPRE

The first web-based multiattribute decision analysis tool

- Super Decisions (Saaty)

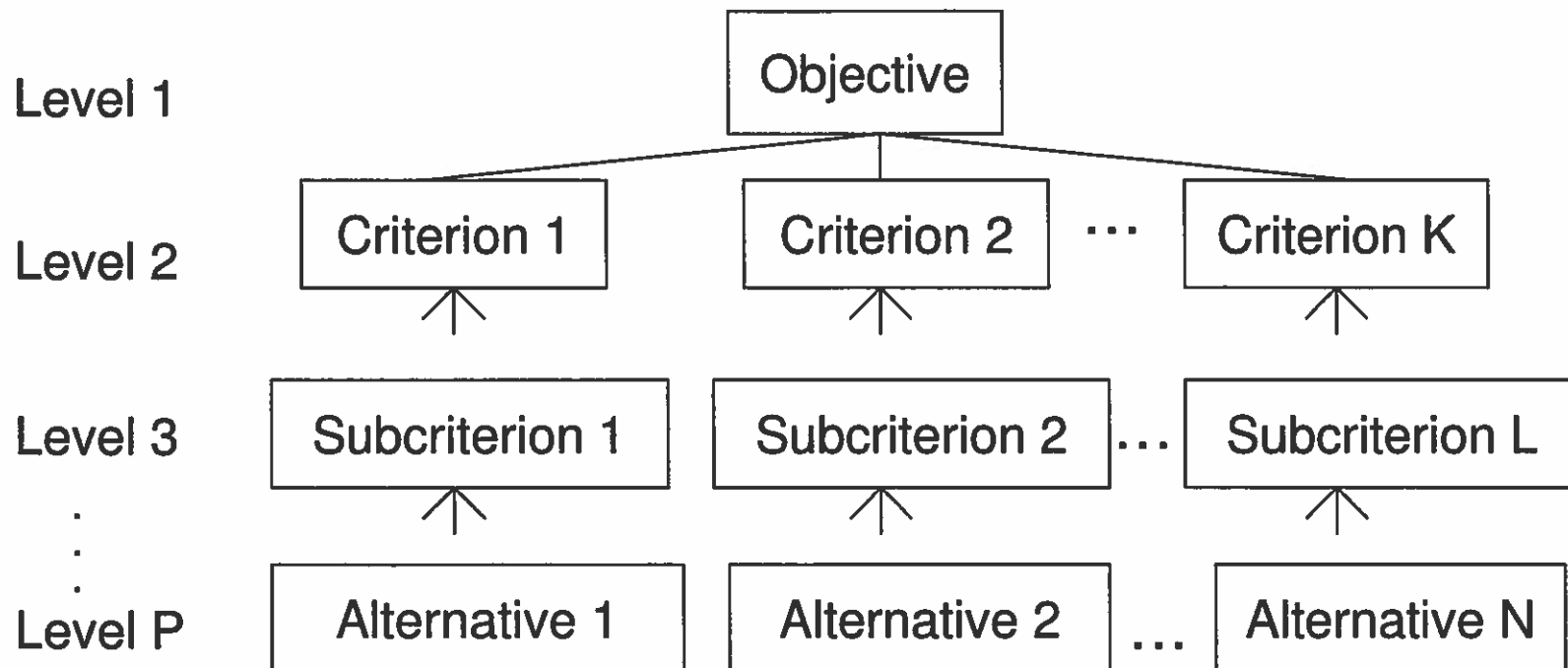
This software implements the analytic network process (decision making with dependence and feedback)

AHP Overview

- Analysis tool that provides insight into complex problems by incorporating qualitative and quantitative decision criteria
- Hundreds of published applications in numerous different areas
- Combined with traditional OR techniques to form powerful “hybrid” decision support tools
- Four step process

The Analytic Hierarchy Process

- Step 1. Decompose the problem into a hierarchy of interrelated decision criteria and alternatives



Hierarchy with P Levels

The Analytic Hierarchy Process

● Illustrative example

Level 1:
Focus

Best Fishery
Management Policy

Level 2:
Criteria

Scientific

Economic

Political

Level 3:
Subcriteria

Statewide

Local

Level 4:
Alternatives

Close

Restricted
Access

Open Access

Partial Hierarchy: Management of a Fishery

The Analytic Hierarchy Process

- Step 2. Use collected data to generate pairwise comparisons at each level of the hierarchy

Illustrative Example

	Scientific	Economic	Political
Scientific	1	a_{SE}	a_{SP}
Economic	$1/a_{SE}$	1	a_{EP}
Political	$1/a_{SP}$	$1/a_{EP}$	1

Pairwise Comparison Matrix: Second Level

The Analytic Hierarchy Process

- Compare elements two at a time
- Generate the a_{SE} entry
 - With respect to the overall goal, which is more important – the scientific or economic factor – and how much more important is it?
 - Number from 1/9 to 9
 - Positive reciprocal matrix

The Analytic Hierarchy Process

- Illustrative Example

	Scientific	Economic	Political
Scientific	1	2	5
Economic	1/2	1	2
Political	1/5	1/2	1

- AHP provides a way of measuring the consistency of decision makers in making comparisons
- Decision makers are not required or expected to be perfectly consistent

The Analytic Hierarchy Process

- Step 3. Apply the eigenvalue method (EM) to estimate the weights of the elements at each level of the hierarchy
- The weights for each matrix are estimated by solving

$$A \cdot \hat{w} = \lambda_{\text{MAX}} \cdot \hat{w}$$

where

A is the pairwise comparison matrix

λ_{MAX} is the largest eigenvalue of A

\hat{w} is its right eigenvector

The Analytic Hierarchy Process

● Illustrative Example

	Scientific	Economic	Political	Weights
Scientific	1	2	5	.595
Economic	1/2	1	2	.276
Political	1/5	1/2	1	.128

Pairwise comparison matrix: Second level

The Analytic Hierarchy Process

- Step 4. Aggregate the relative weights over all levels to arrive at overall weights for the alternatives

